THE THEORY OF COSMIC RAY MODULATION

by

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Abstract

The current state of the theory describing cosmic ray modulation in the interplanetary medium is reviewed. Emphasis is given to the problems of determining the transport coefficient for diffusion in energy and position space and in assessing the importance of particle drift motion in three dimensional modulation models.

Chapter headings are as follows:

1. Introduction

The purpose of this review is to provide a description of the current state of the theory for the main effects of the interplanetary medium on the galactic cosmic ray spectrum. Recent work explaining the three dimensional nature of energetic particle motion in the heliosphere, together with a continued input of interesting experimental results from Pioneer 10 and 11 and Voyagers 1 and 2 makes this task necessary. While we will concentrate on the development of the formal description of the topic of modulation, important experimental data requiring explanation will be mentioned.

Two basic reasons motivate the desire to understand cosmic ray modulation. For some, the interaction of energetic particles with the electromagnetic fields of the interplanetary medium is an example of an astrophysical plasma problem which should be capable of solution, given the comprehensive and complementary field and particle data available. For others, the effect of the interplanetary medium is simply to mask our knowledge of the true interstellar cosmic ray spectra below $\sim 10^{10}$ eV and modulation studies are expected to provide suitable correction factors to near Earth measurements of the charge and energy spectra of cosmic ray nuclei and electrons.

It was only in 1954 that Forbush established the 11-year modulation of the cosmic ray intensity in anti-correlation with solar activity which is now recognised as the chief effect of the interplanetary medium on the galactic spectrum (Forbush, 1954). However, pre-1939 searches for anisotropies associated with the galaxy revealed intensity variations dependent upon solar time which in fact correspond to flow patterns associated with the 11-year modulation. Pomerantz and Duggal (1971) in their review of the solar diurnal anisotropy cite Miehlnickel (1938) as an example of such an observation. Transient change in the level of modulation usually associated with geomagnetic activity and known as the Forbush Decrease was first noticed in the 1930's, e.g. Messerschmidt (1933) and Steinmaurer and Grazidei (1933).

Quantitative theoretical understanding of modulation can take two approaches, both deriving from the work of Parker on the solar wind (Parker, 1958a) which established the supersonic plasma outflow from the sun and the Archimedean spiral field pattern. Parker (1958b) pointed out the effect of scattering irregularities in the solar wind magnetic field would be to cause a tendency for low energy cosmic rays to convect outwards with the flow. The reduction in intensity near the sun would result in a balancing, inward diffusive flux along the spiral field lines, driven by the density gradient. A further cause for reduction in the differential number density of particles is the deceleration brought about by expansion of the solar wind medium. This was first mentioned by Singer et al (1962) in connection with a theory for Forbush decreases, but was subsequently applied to the 11-year cycle modulation by Quenby (1965a) and Dorman (1965). A complete Fokker-Planck equation describing diffusion parallel and perpendicular to the mean field, convection and energy loss due to adiabatic deceleration was then quickly provided by Parker (1965b). The only modification to this equation required by more recent studies is the inclusion of an acceleration term due to the relative motion of waves in the solar wind reference frame (Fisk 1976a). Simplicity in understanding modulation, especially the associated anisotropy, derives from the work of Gleeson and Axford (1968) on the Compton-Getting factor, $C = 1/3U\ \partial/\partial T(\alpha TU)$ for number density U at kinetic energy T with $\alpha = T+2E_o/T+E_o$. $C\underline{V}U$ represents the first order correction to any wind frame anisotropy on transformation to the stationary reference frame with solar wind velocity $\underline{V}$. Hence with a diffusion gradient driven flux purely along the field lines in the wind frame, $K_{\parallel}\ \partial U/\partial s$ for parallel diffusion coefficient $K_{\parallel}$ and arc length along the field s, the total rest frame streaming is

$$\underline{S} = C\underline{V}U - K_{\parallel} \frac{\partial U}{\partial \underline{s}} \tag{1}$$

In equilibrium the radial component of $\underline{S}$ is observed to be roughly zero, so $CVU = K_{rr}\partial U/\partial r$ where K_{rr} is the resolute of $K_{\parallel}$ in the $\hat{\underline{r}}$ or radial direction. Also the azimuthal streaming is $K_{\phi r}\ \partial U/\partial r$ and is from the East since there is no V_ϕ and the Archimedean spiral field lines run east to west as seen by inward moving particles. Because the spiral angle is ≈ 45° at the Earth, we see immediately that the azimuthal streaming giving rise to the solar diurnal variation is ≈ CVN. Also the depth of modulation is given by

$$\frac{U(1\ AU)}{U(r_\infty)} = \exp\left(-\int_{r_1}^{r_\infty} \frac{CV}{K_r}\, dr\right) \tag{2}$$

where r_∞ represents the boundary of the modulation region or solar cavity.

Experimentally it is found that at high magnetic rigidities ($\gtrsim$ 1 GV), we can write

$$\frac{U\ (1\ AU)}{U(r_\infty)} \cong \exp\left(-\frac{M}{P}\right) \tag{3}$$

where P is in GV and independent studies of $U(r_\infty)$ based on spectral composition data and galactic radio emission suggest M ≈ 0.3 - 0.6 GV at solar minimum. No integral of (2) can be performed reliably as yet. Attention has been paid to perpendicular diffusion and drift motion under field gradient and curvature and to out-of-ecliptic effects which can substantially modify the approximation (1) (e.g. Lietti and Quenby, 1968; Jokipii, Levy and Hubbard, 1977).

An alternative viewpoint or approximation to modulation arises from the hypothesis of Ehmert (1960) that the particles move under a heliocentric electric potential. This is not due to an electrostatic charge, as originally postulated, but results from the smooth field limit or Archimedean spiral representation of the IMF (Interplanetary Magnetic Field). Here, the rest frame electric field is $\underline{E} = -\underline{V} \times \underline{B}$ because of the relativistic transformation appropriate to infinite electrical conductivity in the moving frame. For a steady $\underline{B}$ and $\underline{V}$, the electric field can be represented by a potential

$$\psi = -a^2\, \Omega\, B_1^r \sin\lambda \tag{4}$$

where a is the distance from the sun to the earth, B_1^r is the radial component of the magnetic field at a and λ and Ω are respectively solar latitude and angular speed (e.g. Stern, 1964). This representation fails to explain azimuthal streaming because there is no scattering to break Liouville's theorem which says that since there is access from all directions to a given point in the electrostatic potential, no anisotropy can result. It does however provide a rough description of the level of cosmic ray modulation and the possible dominating importance of three dimensional particle drift motion under gradient and curvature forces (Jokipii and Kopriva, 1979; Kota, 1979). These drifts move particles against E to or from the solar polar regions.

From the viewpoint of modulation studies as a means of doing collisionless plasma physics, the detailed understanding of the interaction of cosmic rays with interplanetary waves and discontinuities is important. Jokipii (1966) and Roeloff (1966) pioneered the attack on this problem using the concept of the resonant wave-particle interaction to scattering in pitch angle and hence $K_{\parallel}$ and to estimate also the magnitude of perpendicular diffusion. Acceleration during wave-particle interactions was studied by Tverskoy (1967) who included the effects of both resonant interaction and those of long wavelength fluctuations. An example of a related study on the ability of shock fronts to accelerate a low energy solar particle population is that of Sarris and Van Allen (1974).

Although limitations in the amount of material we can reasonably cover in this review precludes a detailed account of Forbush Decrease theory, we should mention at this point that such investigations also relate to the plasma physics of the interplanetary medium. In this case, it is likely to be the modifications resultant upon the emission of a high speed stream or a blast wave from the sun which causes the event. Theories of the decrease either derive from the disordered field model of Morrison (1956) or an ordered field geometry (Alfven 1954). Most progress has been made based upon a geometry derived from Parker's (1963a) blast wave model which introduced the idea of a moving, leaky barrier.

Whereas it is probably correct to say that almost all the main effects governing the overall cosmic ray modulation were known by 1965, the relative importance of the various terms in the Fokker-Planck transport equation describing the interplanetary propagation and the evaluation of the transport parameters employed remain as topics to be decided. Recent experimental results which profoundly influence the course of the theoretical development of the subject are the continuing small values of the density gradient seen out to 20 AU and beyond by Pioneer 10, the increasing confirmation that the mean IMF is entirely outward or inward above ± 15° → 30° solar latitude, evidence for interplanetary acceleration at low energies and a variety of low energy solar particle and galactic charge composition and flow data which seem to be incompatible with low values of the scattering mean free path derived from local magnetic field data.

Our review must be selective in the experimental data mentioned and our theoretical development will aim to present simple and therefore non-rigorous proofs for the benefit of those who are not well trained in Applied Mathematics. For both these reasons we now list previous reviews, through which a more thorough knowledge of the subject can be obtained. Theory is reviewed in book or journal form by Parker (1963b), Dorman (1963), Quenby (1967), Parker (1969), Jokipii (1971), Volk (1975), Fisk (1979) and Gleeson and Webb (1979) while reviews given at conferences include Parker (1965a), Quenby (1965b), Gleeson (1971), Quenby (1973, 1977), Forman (1975), Jokipii (1979) and Lee (1981). Experimental evidence is reviewed in book or journal form by Webber (1962), Lockwood (1971), Pomerantz and Duggal (1971, 1974) and Moraal (1976), while conference reviews include Webber (1967), Gleeson (1971), Quenby (1973), Moraal (1975), Pomerantz (1975), Nagashima (1977), Duggal (1977), Webber (1979), McKibben (1981) and Somogyi (1981). Related reviews of solar particle propagation include McCracken and Rao (1970), Lin (1974), Palmer (1981) and Quenby (1982). Early work can be followed in Elliot (1952) and Singer (1958).

2. The Interplanetary Magnetic Field and The Solar Wind

Modulation theory can only reasonably be developed within the context of current models for the solar wind, the interplanetary magnetic field and the termination of the heliosphere at the boundary with the interstellar medium. Usually it has been the advance of knowledge concerning the solar plasma configuration which has pre-dated improvement in modulation theory. Since we are limited by current solar wind data, including the three-dimensional field and plasma distribution, it is reasonable to briefly summarise this knowledge before critically reviewing cosmic ray propagation theory.

Starting from the idea of a sphecially symmetric, supersonic plasma outflow from the sun, Parker (1958a) pointed out that at the very high magnetic Reynolds number appropriate to conditions in the interplanetary medium, frozen-in field lines initially radial in direction at some point, r = b, close to the solar surface, would follow the Archimedes spiral

$$\begin{aligned} r &= Vt + b \\ \phi &= \phi_o + \Omega\, t \sin\theta \end{aligned} \tag{5}$$

in a spherical coordinate system with $\theta = 0$ defining the rotational axis of the sun and ϕ_o as the longitude of origin of the streamline or field line at r = b where the flow just becomes supersonic. Conservation of magnetic flux in the diverging geometry requires B_r, the radial component of the magnetic induction (to be known as 'field' from henceforth) to go as $B_r = B_o(b/r)^2$ for $\underline{B} = B_o$ at $r = b$. The streamline/field line makes an angle $\psi = \tan^{-1}(\Omega r \sin\theta/V)$ to $\underline{\hat{e}}_r$ at r and hence $B_\phi = B_o(b/r)^2\, \Omega r \sin\theta/V$.

Modifying Parker's idea to take into account the suggestion of Schulz (1973) and others that the solar field and its extension into the solar wind can be represented by the dragging out of a dipole, tilted at an angle α with respect to the solar rotational axis,

$$\underline{B} = B_o\left(\frac{b}{r}\right)^2 \left[\underline{\hat{e}}_r - \frac{r\,\Omega\,\sin\theta}{V}\right]\left\{1 - 2\,S\left[\theta - \left(\frac{\pi}{2} + \alpha\,\sin\left(\phi - \frac{r\Omega}{V}\right)\right)\right]\right\} \tag{6}$$

where S(X) is the Heaviside step function. The current sheet representing the bounday between inward and outward interplanetary field is given by

$$\theta = \frac{\pi}{2} + \alpha \sin\left(\phi - \frac{r\Omega}{V}\right) \qquad (7)$$

e.g. Jokipii and Kopriva, 1979). Pioneer 11 data (Smith et al, 1978) suggest that $\alpha \simeq 16°$ in 1976 because the sector structure in the field had almost completely disappeared when the spacecraft reached a heliolatitude of 16°.

Figure 1 (Jokipii and Thomas, 1980), shows a computer simulation of the warped current sheet separating the outward and inward regions of field polarity. It is the wobbling of the solar dipole equator with respect to a fixed point in interplanetary space together with the roughly radial motion of the solar wind convecting the field at a finite speed and causing a delay in the appearance of one particular sign of the photospheric field at a point in space that produces the pattern.

The relatively simple tiled dipole model does not, however, satisfy completely the totality of the sector structure data available, in particular on the Rosenberg-Coleman (1969) effect. This measures the dependence of magnetic field polarity on heliolatitude and the problem is discussed by Moussas and Tritakis (1980). Hakamada and Akasofu (1981) for example find two peaks per solar cycle in the maximum heliolatitude of the current sheet between 1965 and 1978, namely in 1968 and 1974 and a maximum tilt angle of 27°. They found this by fitting a latitude dependence to the solar wind speed as revealed by interplanetary scintillation data on the movement of inhomogeneities across radio sources (Dennison and Hewish, 1967; Sime and Rickett, 1978). The angle of dipole tilt was then made to fit the chief feature of the observed solar wind speed at the earth. Polar coronal holes are supposed to be centred on the dipole axis and hence the equatorwards spreading of the polar streams (e.g. Suess et al 1976) appear in the northern and southern hemispheres at longitudes separated by 180°. Hakamada and Akasofu show the solar wind speed to be a maximum twice per 27-day rotation period and in phase with their predicted maximum of solar magnetic latitude. This observation together with the expected sign reversal of $\underline{B}$ each half rotation period fits into the idea of the equatorwards spreading from coronal holes. However to obtain a 4-sector structure as sometimes observed, an additional longitudinal wave structure is postulated for the current sheet, possibly due to for example two northern coronal streamers in 1971-1973 (Howard and Koomen, 1974).

An overall summary of the time variations of the interplanetary scintillation measurements is given by Coles et al 1980, from whom Figure 2 is reproduced. While the equatorial wind speed remains relatively steady, the high latitude speed and positive latitudinal gradient in speed are both greatest at solar minimum. Note also the contraction of the coronal hole area near solar maximum.

Simultaneous Helios I and Helios 2 magnetometer observations between 0.28 and 1 AU confirm the average tilted current sheet configuration giving a angle $\alpha = 10°$ (Villante et al 1979). However observational scatter about the mean result can be interpreted in terms of local distortion to the warped current sheet boundary and Figure 3 is one possible model satisfying the data. Four sectors are only found at northern latitudes in this model.

Concerning the overall and fluctuating behaviour of the magnetic field, a recent study by Thomas and Smith (1980a) confirms the average Parker spiral angle out to 8.5 AU but shows that the field direction exhibits more variability in quiet than in interaction regions of the solar wind. Pioneer 10 data out to 5 AU approximately fit the expected r^{-2} radial magnetic field dependence and the associated r^{-1} azimuthal field variation (Rosenberg et al 1978) though some departure from the latter law due to correlated $\delta V_r \, \delta B_\phi$ fluctuations can be accommodated in the scatter of the results (Goldstein and Jokipii 1977). Thomas and Smith (1980b) have studied the radial power spectrum of fluctuations and Figures 4 and 5 respectively show the power in the transverse and $|B|$ or longitudinal fluctuation components at different radial distances. The spectral indices at high frequencies tend to increase from -1.7 to -1.4 between 1 and 7 AU. Integrated power spectral data show a radial variation proportional to $r^{-3.03}$ for the transverse component and proportional to $r^{-2.04}$ for $|\delta B|$. Thus the scaling of the transverse power is close to that of $|B|^2$, which varies as $r^{-2.74}$ (Rosenberg et al 1978) while the longitudinal fluctuations become relatively more important further out.

Burlaga (1979) has recently reviewed the subject of wave motion within the IMF from the viewpoint of identification of the propagation modes for the disturbances. It is generally agreed that most fluctuations are Alfvénic, obeying $\delta \underline{V} = A \delta \underline{B}$ where $A = V_A/B_o$ for Alfvénic speed V_A and mean field $\underline{B}_o$. However, it is necessary to distinguish between either plane, transverse wave fluctuations with $\delta B(t)$ oscillating in 2 dimensions so that $|\underline{B}|$ = constant and the perturbation vector moves on a small circle of a sphere of this radius but restricted to the plane perpendicular to $\underline{B}$ or alternatively the more general $|\underline{B}|$ = constant wave case where $\delta \underline{B}(t)$ fluctuates over the surface of this sphere in different planes at different positions (Whang 1973, Goldstein et al 1974). In the second case, there is a $\delta B_\perp : \delta B_\parallel$ anticorrelation and observations confirm this latter situation is the most common (Sari 1977). Both types of waves are predicted to follow a $|\delta \underline{B}(r)| \approx r^{-3/2}$ radial dependence (Whang 1973), in agreement with the Thomas and Smith power spectra data. By studying the time it takes these fluctuations to convect in the solar wind past two close spacecraft, it is possible to study the alignment of the minimum variance direction or equivalently the $\underline{k}$ (propagation vector) orientation (Denskat and Burlaga, 1977). This study indicates a tendency for $\underline{k}$ to align with the average field direction, rather than the radial direction. Theoretical predictions in the absence of velocity gradients had suggested that Alfvén waves would have $\underline{k}$ vectors more nearly radial in orientation (Barnes, 1969; Völk and Alpers, 1973). Furthermore, the tendency of the $\underline{k}$ direction to align with $\underline{B}$ includes the leading and trailing edge parts of stream interaction regions contrary to the prediction of some that $\underline{k}$ would point west of the earth-sun line at the front and east of this line to the rear of the interaction.

Large scale discontinuities also exist in the IMF. Discontinuous field changes in direction $\gtrsim 30°$ occur at a rate of 0.5 to one per hour and there are roughly equal numbers which are tangential discontinuities with $\underline{B}$ parallel to the surface and no mass flow across and rotational discontinuities with a field component normal to the surface and mass flow across this plane (e.g. Burlaga et al, 1977). While most rotational discontinuities resemble Alfvénic fluctions in that they conserve $|B|$, changes in the plasma anisotropy, $(P_\perp - P_\parallel)$, across the surface allows possible change in $|\underline{B}|$ (Hudson, 1970). Interplanetary shocks, another discontinuity class, are distinguished by large velocity change and plasma compression. They are either of the transient type where flare accelerated gas pushes its way through the ambient solar wind or of the corotating type associated with fast solar wind stream or interaction regions. In the outer solar system, two basic corotating regions are found, each comprising of a fast and reversed shock pair and following a spiral pattern, with compressed gas inside the regions and rarefied gas and quiet magnetic conditions outside (Smith and Wolfe, 1979). Other possible special field configurations seem to exist occasionally, such as the closed magnetic loops or tightly wound helixes seen by Burlaga and Klein (1980) and others. Interaction regions often seem to overtake and absorb the neutral sheet, separating the inward and outward IMF (Thomas and Smith 1980c). Hakamada and Akasofu (1982) model this and other aspects of the kinematics of the 3-dimensional solar wind disturbance structure.

3. The Fokker-Planck or Modulation Transport Equation

Although the basic equation describing cosmic ray modulation has been known since 1965 (Parker 1965b), there has been a continuing theoretical effort to understand the various terms and refine the derivation. Evolving knowledge of the interplanetary electromagnetic conditions has greatly influenced this process. Basically we may comprehend modulation as the result of a competition. Galactic particles attempt to follow the Archimedean spiral field lines into the sun, but suffer scattering due to magnetic waves. Provided a density gradient is set up, there must be an inward diffusive current, supplemented in principle by transverse scattering and large-scale drift motion across the field lines to or from the polar Archimedean field lines where the intensity may be rather different. This inward current is balanced by an outward convective sweeping as the scattering centres are carried by the solar wind. Since in the solar wind reference frame these scattering centres are receding from each other, a particle energy loss is also expected to deplete the intensity in a differential momentum range.

A tensor diffusion coefficient may be constructed, based on the belief that the Archimedean pattern is dominant and that field fluctuations are relatively small. Note that the power levels for the fluctuations mentioned in section 2 actually imply $\langle\delta B_{\perp}\rangle/B \sim 0.3 \rightarrow 0.5$, so this approximation requires careful investigation. Individual particle motion is considered under the guiding centre approximation and field irregularities are thought to cause appreciable change from the initial helical trajectory only after many gyrations. By working initially in the solar wind frame where the electric field $\underline{E} = -\underline{V} \times \underline{B}$ is generally small, since $V_A/V \gtrsim 1/10$, the elastic collision approximation is useful. The particle kinetic theory approach of Parker (1958a) and the Boltzman equation approach of Axford (1965) and Quenby (1966) to this diffusion tensor can be illustrated in the following manner which simply re-iterates a standard, plasma physical treatment employing a relaxation length (Allis 1956).

Introduce $\underline{v}_g$ as the guiding centre velocity for particle motion in uniform magnetic induction $\underline{B}$, electric field $\underline{E}$ with $\underline{\omega}_b = e\underline{B}/mc$ as the cyclotron angular frequency. Let complete randomisation of individual particle motion occur with frequency ν_c in elastic, "hard sphere" collision. Hence Newton's second law is

$$\dot{\underline{v}}_g = \underline{a} + \underline{\omega}_b \times \underline{v}_g - \nu_c \underline{v}_g \quad (8)$$

with $a = e\underline{E}/m$ the linear acceleration and we take the steady state situation, $\dot{\underline{v}}_g = 0$. Taking the vector product $\underline{\omega}_b$ x (8) we find

$$(\nu_c + \underline{\omega}_b \times)\underline{a} = (\nu_c^2 + \omega_b^2)\, \underline{v}_g - (\underline{\omega}_b \cdot \underline{v}_g)\omega_b \quad (9)$$

which can be written in tensor notation as $v_{gi} = \mu_{ij} E_j$ where μ_{ij} represents the various conductivities. However if we represent the force per unit volume and per unit momentum interval at $\underline{p}$, $Ue\underline{E}$ by -grad P where U(r,p,t) is the differential number density and define a particle pressure $P = 1/3\ Um\ v^2$ at particle velocity $\underline{v}$ in unit momentum interval, we find

$$v_{gi} = -\frac{1}{U} K_{ij} \frac{\partial}{\partial r_j} U \quad (10)$$

For $\underline{r} = \underline{i}x + \underline{j}y + \underline{k}z$ and $\underline{B}$ in the $\underline{k}$ direction,

$$K_{ij} = \frac{v^2}{3} \left\| \begin{matrix} \frac{\nu_c}{\nu_c^2+\omega_b^2} & \frac{-\omega_b}{\nu_c^2+\omega_b^2} & 0 \\ \frac{\omega_b}{\nu_c^2+\omega_b^2} & \frac{\nu_c}{\nu_c^2+\omega_b^2} & 0 \\ 0 & 0 & \frac{1}{\nu_c} \end{matrix} \right\| \quad (11)$$

Hence $\underline{v}_g$ becomes a streaming velocity, driven by a particle density gradient and a parallel mean free path may be introduced, $\lambda_{\parallel} = v\ \nu_c$, appropriate to momentum $\underline{p}$. The parallel diffusion coefficient is $K_{\parallel} = \lambda_{\parallel} v/3$ and when $\nu_c << \omega_b$, the other diagonal terms of the tensor yield diffusion coefficients, $K_{\perp} = v\lambda_{\parallel}/3\omega_p^{\ 2} = \nu_c\, \rho^2/3$ where ρ is the cyclotron radius and we are involved in scattering by a length ρ every $\lambda_{\parallel}$ distance travelled along $\underline{B}$. The off diagonal terms are then analogous to the Hall conductivity and correspond physically to streaming due to a guiding centre gradient perpendicular to $\underline{B}$.

To transform this diffusion anisotropy from the moving or wind frame to the fixed reference frame in which observations are made, the anisotropy must be corrected for the Compton-Getting effect. This factor arises from the bunching of the particle distribution function in the $\underline{V}$ direction and change in the energy of individual particles due to the convective motion. Gleeson and Axford (1967) first obtained the accepted expression for this correction by applying the Boltzmann equation in the rest frame to "hard sphere" scattering in a spherically symmetric wind with radial $\underline{B}$. Forman (1970) has provided a more easily followed proof which depends on the Lorentz invariance of the particle distribution function on reference frame transformation :

$$f(\underline{p}) = f'(\underline{p}') \quad (12)$$

where primed quantities refer to the moving frame. For relative velocity $V \ll v$

$$\underline{p} - \underline{p}' \simeq -\frac{p}{v}\underline{V} \quad (13)$$

and a Taylor expansion of (12) yields

$$f(\underline{p}) = f'(\underline{p}) - \frac{p}{v}\underline{V} \cdot \nabla_p f'(p) \quad (14)$$

for a gradient ∇_p in momentum space. If $\underline{n}$ is the unit vector in the anisotropy direction in the stationary frame,

$$\nabla_p f' = \underline{n} \frac{\partial t}{\partial p} + 0(\frac{V}{v}) \quad (15)$$

as the anisotropy in the primed frame is of order V/v and so

$$f(\underline{p}) = f'(\underline{p}) - p \frac{df}{dp} \underline{V} \cdot \frac{\underline{\underline{n}}}{v} + 0(\frac{V}{v})^2 \quad (16)$$

and the first term in (16) is the Compton-Getting correction to the anisotropy. Going from gradient of the distribution function to differential number density in kinetic energy, T, yields the streaming correction

$$C \underline{V} U = (1 - \frac{1}{3U} \frac{\partial}{\partial T} \alpha T U) \underline{V} U \quad (17)$$

where $\alpha = T + 2E_O/T + E_O$ and E_O is rest energy. For a spectral index $n = - \partial \ell nJ/\partial \ell nT$ for differential intensity $J(\underline{r},T) = v U/4\pi$, $C = (2 + \alpha n)/3$. It may be shown (Dorman et al 1977) that the kinetic equation developed by Dolginov and Toptygin (1968) by starting with the Liouville equation and a spectrum of plane wave turbulence leads ultimately to a similar result to (17). Dorman et al also show the equivalence of the Dolginov and Toptygin work to the Fokker Planck of Parker (1965b) in their work which was originally published in a Russian journal in 1966.

Our next step in setting up the complete transport equation is to consider diffusion in energy space. Parker (1965b) and Jokipii and Parker (1967, 1970) used an adiabatic deceleration rate

$$\langle\dot{p}\rangle_{ad} = -\frac{p}{3}\frac{\partial}{\partial \underline{r}} \cdot \underline{V} \quad (18)$$

true either in the fixed or the wind frame. An alternative and more physically understandable approach is to consider energy change in the fixed frame. There must be a current $\partial/\partial T$ (dT/dt) U in energy space where (dT/dt) is the mean rate of increase of the particle kinetic energy with respect to time. We may suppose that the solar wind does work against a pressure gradient of the cosmic ray gas. That is, the magnetic field lines are forcing their way past an existing gas distribution. The gas pressure, assumed to be isotropic, is $\alpha TU/3$ and hence the cosmic rays actually <u>gain</u> heat (random) energy at a rate

$$(\frac{dT}{dt}) U = \underline{V} \cdot \text{grad } P \text{ per unit volume, per second}$$

(Fisk 1974, Quenby 1973, Gleeson and Webb, 1974). Thus

$$(\frac{dT}{dt})\ U \quad = \quad V \frac{\partial}{\partial r}\ (\frac{\alpha TU}{3}) \tag{19}$$

At this stage it is appropriate to gather up the previous terms into the Fokker Planck of Parker by setting down the complete continuity equation for $U(\underline{r},T,t)$. Net streaming in $\underline{r}$ and T space with all quantities measured in the stationary frame is

$$\frac{\partial U}{\partial t} + \text{div}\ \underline{S} + \frac{\partial}{\partial T}\ (\frac{dT}{dt})U \quad = \quad 0 \tag{20}$$

where $\underline{S} \; = \; C\ \underline{V}\ U - \underline{\underline{K}}\ .\ \nabla\ U$ (21)

is Compton Getting plus diffusive streaming. Substituting (19) and (21) into (20) for a spherically symmetric situation with no statistical acceleration yields

$$\frac{\partial U}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}\ (r^2\ VU - r^2\ K_{rr}\ \frac{\partial U}{\partial r}) \quad = \quad \frac{1}{3r^2}\frac{\partial}{\partial r}\ (r^2\ V)\ \frac{\partial}{\partial T}\ (\alpha TU) \tag{22}$$

(cf. Parker 1965b, Axford and Gleeson 1967, Skilling 1975).

Webb and Gleeson (1979) have provided further insight into the derivation of this equation (22) employing the adiabatic loss (18) by first showing three different ways in which (18) can be established. Then a procedure given by Jokipii and Parker 1970 is followed to yield (22). The first of Webb and Gleeson's approaches is to take a continuity equation similar to (20) but written in terms of U^*_p which is particle density measured with respect to moving frame momentum, p', and fixed frame position, r. Likewise $\underline{S}^*$ is defined and shown to be $\underline{S}^* = \underline{V}\ U'_p + \underline{S}'_p$ where U'_p and $\underline{S}'_p$ are moving frame quantities. The momentum change term $\langle\dot{p}'\rangle^*$ analogous to dT/dt in (20) was evaluated by noting that momentum can change because of the Lorentz force or as a consequence of the spatial and time dependence of the solar wind velocity. Averaging over a group of nearly isotropic particles and using the Lorentz transformation yielded

$$\langle\dot{p}'\rangle^* \quad = \quad \frac{p'}{3}\frac{\partial}{\partial \underline{r}}\ .\ \underline{V} \tag{23}$$

algebraically identical to Parker's form. It was noted that the derivation did not rely on the detailed form of the Lorentz force provided wave motion in the wind frame was neglected.

The second derivation followed Skilling (1971) in taking moments of the ensemble averaged Liouville equation. Again it was noticed that the momentum change terms in the resulting continuity equation depended only upon $\partial/\partial\underline{r}.(\underline{VF})$ and not on the details of the Lorentz force (F is the fixed frame distribution function).

Webb and Gleeson's third approach was to re-do the original work on betatron and inverse-Fermi deceleration first mentioned by Singer et al (1962) and used by Quenby (1965a, 1967). In the solar wind frame, cosmic rays lose energy as they bounce backwards and forwards along the spiral IMF lines because of the net recession of the scattering centres. This inverse of the Fermi acceleration process is shown to give

$$\langle \dot{p}' \rangle_{IF} = -\frac{p'}{3} \cos^2 \psi \, \frac{\partial V}{\partial r} \qquad (24)$$

where ψ is the angle between the field and the radial direction. Also in the wind frame there is a betatron deceleration. From the viewpoint of a reference point moving with the wind, there is a finite curl $\underline{E}$ because of the expansion of the surrounding plasma. Hence each gyro-period the particle loses energy because $\partial \underline{B}'/\partial t$ is negative. It is found that

$$\langle \dot{p}' \rangle_B = -\frac{p'}{3} [\nabla . \underline{V} - \underline{B} \frac{\underline{B} : \nabla \underline{V}}{B^2}] \qquad (25)$$

which is algebraically the adiabatic rate together with a second term dependent on field direction. (24) plus (25) together yield (23) which is the adiabatic rate purely measured in the moving frame. Thus it seems that (18) represents average deceleration with momentum measured in the moving frame and position in either the fixed or moving frame. The model used in the above, third derivation is more specific than employed in the previous two cases, but nevertheless greater physical insight is obtained into the real situation.

Adopting (23) and the continuity equation of the first method yields the transport equation

$$\frac{\partial U^*_p}{\partial t} + \frac{\partial}{\partial \underline{r}} \cdot [\underline{V} \, U^*_p - \underline{\underline{K}} \cdot \frac{\partial U^*_p}{\partial \underline{r}}] + \frac{\partial}{\partial p'} [- \frac{p'}{3} \frac{\partial}{\partial \underline{r}} \cdot \underline{V} \, U^*_p] = 0 \qquad (26)$$

Jokipii and Parker (1970) and Webb and Gleeson (1979) show $U_p = U^*_p + O(V/v)^2$ so on relabelling the momentum p' by p, the familiar momentum form of (22) is obtained.

$$\frac{\partial U_p}{\partial t} + \frac{\partial}{\partial \underline{r}} \cdot (\underline{V} \, U_p - \underline{\underline{K}} \cdot \frac{\partial U_p}{\partial \underline{r}}) - \frac{1}{3} \frac{\partial}{\partial \underline{r}} \cdot \underline{V} \frac{\partial}{\partial p} (p \, U_p) = 0 \qquad (27)$$

From the standpoint of this derivation, there is energy loss on average for all particles in the wind frame and the fixed frame equation arises finally as a result of the transformation of number density. In our previous derivation leading to (22) we kept the number density in the fixed frame where heating occurred for the average particle. However an energy loss term appeared and dominated the final equation via the Compton Getting transformation. Physically this may correspond to the attempt by the solar wind to remove particles from the observer via the outward convection process.

It is now necessary for completeness and to correspond to current knowledge to add terms to the Fokker Planck which allow for the acceleration in the solar wind by mechanisms related to the Fermi statistical acceleration of cosmic rays. The interaction under discussion is the statistical energy increase when charged particles scatter off waves moving in the wind frame. As in the original Fermi situation, a mean acceleration, $\langle \Delta T \rangle$, is possible, averaged over all head-on and tail-on collisions and also a statistical, energy diffusion term, $\langle \Delta T^2 \rangle$, due to fluctuations in the energy gain per collision (Davis, 1956). Hence it is possible to write the continuity equation in energy space as

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial T} \left(D_T U - \frac{\partial}{\partial T} D_{TT} U \right) = 0$$

However, provided Liouville's theorem holds for particle trajectories in the IMF in a fine grained sense, Dungey's (1965) proof for spatial diffusion that the mean and root mean square diffusion coefficients can be related may be adopted to the energy case and both terms can be combined in the form $p^{-2}\, \partial/\partial p\, p^2\, D_{pp}\, \partial f/\partial t$ for the divergence of the distribution function current (Moussas et al 1982a). Transformation from f(p) to U(T) yields $D_{TT} = 2TD_T$ and

$$\frac{\partial U}{\partial t} = - \frac{\partial}{\partial T} \left(\frac{D_{TT}\, U}{2}\right) + \frac{\partial}{\partial T} \left(D_{TT} \frac{\partial U}{\partial T}\right) \tag{28}$$

for the statistical acceleration term (Fisk 1976a, Wibberenz and Beuermann, 1972). Hence the Fokker Planck with spherical symmetry is now

$$\frac{\partial U}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \left(VU - K_{rr} \frac{\partial U}{\partial r}\right) = \frac{1}{3r^2} \frac{\partial}{\partial r} (r^2 V) \frac{\partial}{\partial T} (\alpha T U)$$

$$+ \frac{\partial}{\partial T} \left(\frac{D_{TT}\, U}{2}\right) - \frac{\partial}{\partial T} D_{TT} \frac{\partial U}{\partial r} \tag{29}$$

4. Derivation of the Transport Coefficients

The next step in the description of modulation theory is to consider the magnitude of the various transport parameters or diffusion coefficients in (29). In principle, these may be derived from a precise knowledge of the interplanetary magnetic and electric field values and their fluctuation. In practice, there are problems both in the theory relating the local field values to the transport parameters because the fluctuations are so large and also because the spatial dependence of these fluctuations is incompletely known for the whole solar cavity.

4.1 The Parallel Diffusion Coefficient

The lowest order approximation to particle motion in interplanetary space at rigidities for which $\rho \ll r$ (cyclotron radius small compared with scale of medium) is that the guiding centres follow the field lines. Diffusion parallel to $\underline{B}$ is then caused by sudden or progressive change in the particle pitch angle θ resulting in motion past $\theta = 90°$. All analytical theories for the parallel mean free path, $\lambda_{\parallel}$, start from the idea of Doppler-shifted gyroresonance. Near relativistic particles see an essentially stationary distribution of waves of different frequencies but they interact preferentially with those whose wavelengths λ_W match the spatial distance over which the particles make one gyroperiod. Thus the resonance condition is $k = 2\pi/\lambda_W = \omega_b/v_{\parallel}$. The derivation of the diffusion coefficient given by Jokipii (1966, 1967), Roelof (1966) and Hasselmann and Wibberenz (1968) is best illustrated in a simple way by following the Kennel and Petschek (1966) formulation given in the context of magnetospheric particle scattering.

Now $\tan\theta = v_{\perp}/v_{\parallel}$ for $v_{\perp}$ and $v_{\parallel}$ as the perpendicular and parallel components of particle velocity. Assume a small perturbation

$\Delta\theta$ from the nearly helical trajectory around a field consisting of $\underline{B}_o$ plus perturbation $\underline{b}(\underline{r})$ due to a transverse wave. Hence

$$\sec^2 \theta d\theta = \frac{\Delta v_\perp}{v''} - \frac{v_\perp}{{v_{\parallel}}^2} \Delta v_{\parallel} \tag{30}$$

Work in the reference frame of the wave so that there is no change in total energy of the particle. In fact we assume the wave to be stationary during the wave-particle interaction. Differentiating $v^2 = {v_\perp}^2 + {v_\parallel}^2 = \text{const.}$ allows us to find from (30), $\Delta\theta \approx -\Delta v_\parallel / v_\perp$. Now if $\Delta v_\parallel$ is due to the wave and particle being in resonance for a time Δt

$$\Delta\theta \approx -\frac{\Delta v_\parallel}{v_\perp} \approx \frac{e v_\perp\, b\, \Delta t}{m c v_\perp} \tag{31}$$

or $\Delta\theta \approx \omega_b\, b/B\, \Delta t$. In practice, no particle is exactly in phase with the wave and the rate of change of relative phase is

$$\frac{d\phi}{dt} = k v_\parallel - \omega_b$$

for wave number k. Adopt the simple criterion that in the resonance time Δt the wave is within $\pm\, \Delta k/2$ of resonance and the phase difference $\Delta\phi \lesssim 1$ radian. Then

$$\Delta t \approx \frac{2}{\Delta k v_\parallel} \tag{32}$$

If we interpret $\lambda_\parallel$ as being the distance in which N separate wave-particle interactions bring about a total pitch angle change of one radian, $\sqrt{N}\, \langle\Delta\theta\rangle \approx 1$ and $\langle v_\parallel\rangle\, N\, \Delta\tau = \lambda_\parallel$ in time $\Delta\tau$. The spatial wave number k is related to a stationary spacecraft observation of waves convected past by $\underline{V}$ at frequency ν by $k = 2\pi\nu/V$. If $b^2/\Delta\nu = P(\nu)$ is the power spectrum of the transverse waves in interval Δk, (31) finally becomes

$$\lambda_\parallel = \frac{VB^2}{4\pi\, P(\nu)} \frac{1}{\nu^2} \tag{33}$$

where $\lambda_\parallel$ corresponds to a particle magnetic rigidity $R = VB/2\pi\nu\; v/\langle v_\parallel\rangle$.

Above we have assumed that a Fokker-Planck equation correctly described parallel diffusion. A more rigorous approach is to derive the Fokker-Planck from the Liouville equation. This desirable aim faces problems in the approximation involved, but nevertheless we outline the treatment, following Jokipii's 1971 review version. Start with the wind frame Liouville equation, neglecting additional, fluctuating electric fields, so that the Lorentz force is

$$\frac{d\underline{p}}{dt} = e(\underline{v} \times \underline{B})$$

For a distribution function $f(\underline{r}, p, t)$,

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \frac{\partial f}{\partial \underline{r}} + e\,(\underline{v} \times \underline{B}) \cdot \frac{\partial f}{\partial \underline{p}} = 0 \tag{34}$$

Let the fluctuations in f and $\underline{B}$ be represented by $f = \langle f\rangle + \delta t$ and $\underline{B} = \langle\underline{B}\rangle + \delta\underline{B}$ and taken an ensemble average,

$$\frac{\partial\langle f\rangle}{\partial t} + \underline{v} \cdot \frac{\partial\langle f\rangle}{\partial \underline{r}} + e(\underline{v} \times \langle\underline{B}\rangle) \cdot \frac{\partial\langle f\rangle}{\partial \underline{p}} = -e\underline{v} \times \langle\delta\underline{B} \cdot \frac{\partial \delta f}{\partial \underline{p}}\rangle \qquad (35)$$

Subtract (35) from (34)

$$\frac{\partial(\delta f)}{\partial t} + \underline{v} \cdot \frac{\partial(\delta f)}{\partial \underline{r}} + e(\underline{v} \times \langle\underline{B}\rangle) \cdot \frac{\partial(\delta f)}{\partial \underline{p}} = -e(\underline{v} \times \delta\underline{B}) \cdot \frac{\partial\langle f\rangle}{\partial \underline{p}}$$

$$- e(\underline{v} \times \delta\underline{B}) \cdot \frac{\partial(\delta f)}{\partial \underline{p}} + e\underline{v} \times \langle\delta\underline{B} \cdot \frac{\partial(\delta f)}{\partial \underline{p}}\rangle \qquad (36)$$

This may be integrated along a particle trajectory U(t') from the initial conditions $f(\underline{p}_o, \underline{r}_o, t_o)$ to $\underline{p}, \underline{r}, t$.

$$\delta f(\underline{p},\underline{r},t) = \delta f(\underline{p}_o,\underline{r}_o,t_o) - e\int_{t_o}^{t} \underline{v} \times \left\{\delta\underline{B}\cdot\frac{\partial\langle f\rangle}{\partial \underline{p}} + \delta\underline{B} \cdot \frac{\partial(\delta f)}{\partial \underline{p}} - \langle\delta\underline{B} \cdot \frac{\partial(\delta f)}{\partial \underline{p}}\rangle\right\}_{U(t')} dt \qquad (37)$$

At this stage, the terms $\delta B\ \delta f$ are usually ignored in relation to $\delta\underline{B}\ \langle f\rangle$. The quasi-linear approximation is then invoked. This assumes that the particles follow helical trajectors along the mean field and that any series expansion involved in evaluating the integral can be rapidly terminated. Klimas and Saudri (1971), state that the basic point concerning the second assumption lies in that the particle gyroradius must be much greater than the typical length over which the field fluctuations are correlated. Hall and Sturrock (1967) show that (37) can be reduced to the Fokker Planck incorporating pitch angle and perpendicular diffusion with coefficients reducible to Jokipii's quasi-linear result (Jokipii 1966). These authors also compute diffusion in energy space. The neglect of some terms in $\delta\underline{B}$ may be unjustified if $\delta B \sim \langle B\rangle/2$ and as we shall mention later, the approximation of helical trajectories is artificial at large θ and again may fail for δB sufficiently large.

The Fokker Planck derivable from quasi-linear theory in cosine pitch angle space ($\cos\theta = \mu$) and in the x and y spatial coordinates for $\langle\underline{B}\rangle$ parallel to the z axis is

$$\frac{\partial f}{\partial t} = -\mu v\frac{\partial f}{\partial z} + \frac{1}{2}\frac{\partial}{\partial \mu}\left[\frac{\langle(\Delta\mu)^2\rangle}{\Delta t}\frac{\partial f}{\partial \mu}\right] + \frac{1}{2}\frac{\partial}{\partial x}\left[\frac{\langle(\Delta x)^2\rangle}{\Delta t}\frac{\partial f}{\partial x}\right]$$

$$+ \frac{1}{2}\frac{\partial}{\partial y}\left[\frac{\langle(\Delta y)^2\rangle}{\Delta t}\frac{\partial f}{\partial y}\right] \qquad (38)$$

$\langle(\Delta x)^2\rangle/\Delta t$ and $\langle(\Delta y)^2\rangle/\Delta t$ express perpendicular diffusion, but at the moment we are concerned with parallel diffusion. Jokipii (1966) obtained

$$\frac{\langle(\Delta\mu)^2\rangle}{2\Delta t} = \frac{(1-\mu^2)}{2|\mu|v}\frac{e^2V}{m^2c^2}P_{xx}\ \left(\nu = \frac{V\omega_b}{2\pi\mu v}\right) \qquad (39)$$

for $m = \gamma m_o$, $\omega_o = e\langle B\rangle/mc$ and $P_{xx} = \delta/\nu^n$ as the power in one perpendicular component of the field fluctuation when ν runs from $-\infty$ to $+\infty$.

From (34) $K_{\parallel}$ is derived by considering the relaxation of a near isotropic pitch angle distribution according to (38) (Jokipii 1966, Hasselmann and Wibberenz 1970, Earl 1974). It is found that

$$K_{\parallel} = \frac{v^2}{2} \int_{-1}^{1} \left[\int_{o}^{\mu'} \frac{(1-\mu^2)}{\frac{\langle(\Delta\mu)^2\rangle}{\Delta t}} d\mu \right] \mu' d\mu' \qquad (40)$$

$K_{\parallel}$ may be estimated experimentally (see Palmer 1982 for a recent review) from the time of arrival and shape of the arrival profile of solar protons, released in a 'prompt' event and propagating past the Earth and other spacecraft positions. These observations are compared with Fokker Planck solutions employing a range of transport parameters. Many studies (Wibberenz et al 1970, Quenby and Sear 1971, Zwickl and Webber 1978, etc.) find $\lambda_{\parallel} \gtrsim 0.05$ AU at $\gtrsim 1$ GV while use of (39), (40) and experimental values of P_{xx} yield $\lambda_{\parallel} \approx 0.01$ AU at these rigidities. Furthermore, solution of the steady state modulation equation in conjunction with measured interplanetary gradients in the cosmic ray intensity also suggest $\lambda_{\parallel} \gtrsim 0.1$ AU (e.g. Lezniak and Webber 1973). A possible solution for this discrepancy between quasi-linear theory and experiment lies in the addition of the focussing term $1/2 \; 1/B \; \partial B/\partial z \; (1-\mu^2) \; \partial f/\partial\mu$ in (38) due to the divergence of the interplanetary field lines (Roelof 1969). However, Earl 1981 finds that if $\lambda_{\parallel}/L \gtrsim 0.1$ where L is the scale of the field variation (1 AU for near earth observations) diffusion rather than adiabatic focussed propagation dominates. This conclusion is confirmed by the computations of Kota et al (1982) who show that Fokker Planck equation solutions without a focussing term remain valid for the expected range of $\lambda_{\parallel}$ values. Palmer (1982) and Quenby (1982) conclude that scatter-free solar particle events at low energies represent only a small sub-set of the totality of events and hence the above experimental-theoretical discrepancy must be taken seriously.

Jokipii (1968), Earl (1974) and Earl and Bieber (1977) draw attention to the lack of scattering at $\theta \to 90°$ according to quasi-linear theory because there are very few high frequency waves for the particles to resonate with at these pitch angles. This apparent cause for persistent anisotropies in the solar proton flux at low θ does not take into account the breakdown of quasi-linear theory at $\theta = 90°$. In fact the propagator U(t') in (37) cannot represent a helical trajectory. If it could, particles would spiral a long time at the same μ value, meeting no waves in gyroresonance. In practice, particles must meet changes in $|\underline{B}|$ which will cause mirroring according to the preservation of the first adiabatic invariant, as pointed out by Quenby et al (1970).

Jokipii (1974) argues that quasi-linear theory can be used if the value of μ taken is that with respect to the total magnetic field, average plus deviation. His formulation depends on the particle making many gyrations before becoming significantly disturbed, which may not be true in reality. Jokipii and Jones (1975) identify the problem that the difference $\mu-\mu_A/\mu_A$ where μ is actual cosine pitch angle and μ_A refers to the angle with respect to the average field may become arbitrarily large as $\mu \to 0$, due to the large finite rotations of the Alfvénic fluctuations and discontinuities encountered in the IMF.

A number of attempts to represent actual trajectories near 90° have appeared in the literature. Physically they all seem to reduce to taking into account mirroring in change $\delta/\underline{B}/(z)$. Mathematically they are attempts to improve our knowledge of the operator $U(t,\tau)$ in

$$D_{\mu\mu} = \frac{<(\Delta\mu)^2>}{2\Delta t} \alpha \int d\tau <\delta B(z)\ U(t,\tau)\ \delta B(z)>$$

which can be obtained by substitution of (37) into (35). Völk (1973) uses a $U(t,\tau)$ which propagates $\delta\underline{B}(\underline{z})$ in the autocorrelation of the field along a statistically scattered set of orbits. He uses a slab model, that is to say $\delta\underline{B}$ is only a function of z (along <B>). Völk is more confident in a simple heuristic procedure which fills in the region $-\mu^* \leq \mu \leq \mu^* = <\delta\underline{B}^2>^{\frac{1}{2}}/\sqrt{2}\ <\underline{B}>$ with the constant value $D^{QL}_{\mu\mu}(\mu=\mu^*)$ where $D^{QL}_{\mu\mu}$ is the quasi-linear diffusion coefficient. The idea here is that the region around $\mu = 0$ is rather uniformly filled by trajectories suffering large scale scatterings. Figure 6 shows the form of $D_{\mu\mu}$ adopted by Völk.

Jones et al (1973) take a partially averaged field trajectory. That is, the field is that which results from averaging over all members of the statistical ensemble with the same value $\delta\underline{B}(z)$ at the field point z. It is assumed that Gaussian statistics hold and

$$\underline{B}_p\ (z',z) = <\underline{B}> + \delta\underline{B}(z)\ .\ \underline{C}(z'-z)$$

where C is the normalised correlation tensor for the fluctuating magnetic field. The orbit is solved from the Lorentz equation and a statistical average is made over all the distribution of $\delta\underline{B}(z)$ values. If the turbulence is 3-dimensional, numerical integration of the motion is necessary. One result of this calculation is a pronounced peak in $D_{\mu\mu}$ at $\mu = 0$ with a width roughly equal to $<\delta B^2>^{\frac{1}{2}}/<\underline{B}>$. Fisk et al (1974) also obtain a δ-function at 90° in the scattering which they ascribe to a resonance with a magnetosonic mode connected with mirroring.

Goldstein (1976) in an approach similar to that of Völk included an extra term in $\Delta p_{\perp}$ in the expression for $D_{\mu\mu}$ which is again connected with mirroring. It leads to important differences in predictions from those of the slab model and in the scaling for different values of $<\delta\underline{B}>/<\underline{B}>$ and the correlation length:gyroradius ratio for gaussian correlation realisation of field fluctuations. In a more recent work, Goldstein (1980) realises that all of the above corrections to quasi-linear theory increase the pitch-angle scattering near 90° and hence decrease the value of $K_{\parallel}$, contrary to what seems to be necessary from observation. His solution to the dilemma is based on the experimental evidence that most IMF turbulence, is Alfvenic, preserving field magnitude. For this mode of turbulence, particle propagation tends essentially to zero at $\mu = 0$ according to Kilmas et al (1977). Goldstein then uses the few percent ($\sim$ 6%) part of the fluctuating field due to $|\underline{B}|$ changes in conjunction with the work of Kilmas and Sandri (1973) on the Landau resonance (compressive mode or mirror effect) at $\mu = 0$ to find $\lambda_{\parallel} \sim 0.3$ AU, independent of rigidity.

In the context of Goldstein's (1980) work it is interesting to note that Webb and Quenby (1974) have tackled the problem of the scattering due to Alfvenic discontinuities preserving $|\underline{B}|$ by a numerical technique. Both they and Hudson (1974) demonstrate that particles are reflected by such a rotational discontinuity, the reflection coefficient for particle flux being $\sim$ 0.065 for an angular change in $\underline{B}$ of 45° across the interface. This reflection coefficient applies to a range of particle cyclotron radii large compared with the scale size of the discontinuity change but small compared with the inter-discontinuity distance. Resonance scattering by a train of discontinuities is also investigated by Webb and Quenby whose final conclusions are qualtitatively in agreement with those of Goldstein (1979). The former authors find that the mean free path when the cyclotron radius is equal to the inter-discontinuity distrance is at least one order of magnitude larger than that given by an application of quasi-linear theory to this 'wavelenth' scale.

Attention has rightly been given to the relationship between scattering theory and the expected and actual form and radial dependence of the IMF fluctuations. Morfill (1975) has provided general expressions based upon quasi-linear theory, which can take into account an arbitrary distribution of $\underline{k}$ vectors of Alfven waves with respect to the mean IMF direction. Following this, Morfill et al (1976) consider two extreme cases; one with $\underline{k} \parallel \hat{r}$ which results from WKB wave propagation theory and the other with $\underline{k} \parallel \langle\hat{B}\rangle$, which tends to be supported by observation (Section 2). These authors adopt the Völk (1973) correction to quasi-linear theory and also take into account medium scale fluctuations in the IMF direction. This last arises because the actual path along the wavy field lines is longer than that following the idealised spiral. For the radial dependence of the waves, the work of Völk and Alpers (1973) was employed. $K_{\parallel}$ turns out to be roughly independent of r at large r although there is a minimum at $r \sim 0.2$ AU. However to fit with spherically symmetric modulation theory and particle gradient data it is found that $\underline{k} \parallel \hat{r}$ rather than $\underline{k} \parallel \hat{B}$ is required. Skadron and Hollweg (1976) used WKB plus quasi-linear theory to demonstrate a small decrease in K_{rr} from 0.1 to 1 AU. However to obtain the near r independence of K_{rr} at $r >> 1$ AU as suggested by the solar particle analysis of Hamilton (1977) and Zwickl and Webber (1977) they require the Alfven wave vectors to be scattered by plasma density fluctuations. Otherwise the tendency for the waves to propagate radially renders them ineffective at particle scattering because waves at a given frequency resonate with higher energy particles due to the effect of projection of the disturbance profile on to the inclined $\langle\hat{B}\rangle$ direction (Morfill 1975). Differences in the basic diffusion coefficients employed may account for the differing conclusions of Morfill et al and Skadron et al.

Analysis of cosmic ray density gradient data by Hsieh and Rickter (1981) and the numerical Fokker-Planck integration of Cecchini et al (1980) in conjunction with solar particle data both support a diffusion coefficient dependence $K_{rr} \propto r^{-2}$ near the sun but nearly independent of r for $r >> 1$ AU. WKB theory which predicts $\langle\delta B_{\perp}^{2}\rangle \propto r^{-3}$ seems consistent with the Thomas and Smith (1980b) analysis but does not give the observed lining up of $\underline{k}$ and $\underline{B}$. In this last respect, the work of Skadron and Hollweg on wave scattering may help.

We see there are problems both with a suitable analytical theory for $K_{\parallel}$ and in predicting the wave evolution with distance. An alternative approach lies in building upon the numerical work pioneered by Jones et al (1973). These workers found $D_{\mu\mu}$ by numerically integrating particle trajectories in a field model in which space is divided into layers perpendicular to the mean field direction with the field in each layer given by the mean plus a transverse perturbation value. The series of perturbations were defined employing an exponential correlation function. Results on an isotropic particle injection distribution are plotted in μ-space. A steady state is set up with particles injected at one pitch angle and removed from the model at two other pitch angles, located one either side of the injection point. $D_{\mu\mu}$ is inversely proportional to the slope of the steady state distribution. Kaiser (1975) and Gombosi and Owens (1979) extended this work including a range of values for $<(\delta \underline{B})^2>^{\frac{1}{2}}/<\underline{B}>$

Moussas et al (1975 and 1978) modified the Jones et al method by defining pitch angle with respect to the local rather than the mean field to be more in accord with experimental data on pitch angle distributions of solar particles. Furthermore these authors used spacecraft magnetometer data to define the field perturbations in each layer and generalised the method to include longitudinal as well as transverse perturbations. Although the model cannot necessarily reproduce the field variation a particle seen going down a magnetic flux tube - a universal defect of all theories mentioned - nor the variation of $\delta \underline{B}$ with position in a direction perpendicular to $<\underline{B}>$, it does deal with finite and discontinuous field changes. Analytical theory clearly has difficulty with such field variations because of its perturbation expansion approach. Any special geometry in the dis-bribution of $\underline{k}$ vectors or mode of polarisation in the IMF waves is automatically taken care of in the model, provided representative data at different radial distances is used. Thus the Goldstein et al (1981) suggestion that the field and particle helicity are in anti-phase to that required by gyro-resonance is incorporated by virtue of the way the field data is employed.

A common conclusion of all the numerical investigations is that when a field defined by the exponential correlation function with small transverse fluctuations alone is used, quasi-linear theory holds as $\mu \to 1$ but breaks down as $\mu \to 0$. The generalised resonance broadening theory of Goldstein (1976) is particularly successful in scaling results at $\mu = 0$. As the ratio $<\delta \underline{B}>/<\underline{B}> \to 1$, there is some conflict in the conclusions. Moussas et al 1975 (see results cited by Forman 1975) find $D_{\mu\mu}$ exceeds the quasi-linear value at large μ by a factor ≈ 2. Kaiser et al (1978) appear to indicate similar large discrepancies although since $D_{\mu\mu}$ was only computed close to $\mu = 0$, it is difficult to be sure. Gombosi and Owen (1979) found agreement at these large field deviations within 20% when $K_{\parallel}$ was deduced from the numerical and quasi-linear theoretical formulations.

Moussas et al 1982b used data at 1 and 5 AU and did computations at different energies to find $K_{\parallel}$ according to real field magnetometer data. They found that between 1 and 100 MeV, $\lambda_{\parallel} \approx 0.03$, roughly independent of energy. Also $\lambda_{rr} \approx 0.01$ AU, roughly independent of distance, provided perpendicular diffusion obtained by similar numerical methods is incorporated. The quasi-linear 1 AU prediction for $\lambda_{\parallel}$ is nearly a factor 3 lower than the numerical value. These authors conclude that although employment of trajectory computations in a model derived from real field data has gone some way to removing the theory-experiment discrepancies in $\lambda_{\parallel}$, a reasonable fit between the two still requires $\lambda_{rr} \propto r^{-2}$ at $r < 1$ AU as suggested by Cecchini et al (1980).

Finally we remark that our discussions so far have been confined to the regime $R \lesssim 1$ GV. Jokipii (1967) has provided one of the few theoretical treatments at higher rigidities where numerical work and detailed experimental checks are both harder to perform.

4.2 Perpendicular Diffusion

Particles may move perpendicular to the mean Archimedes IMF lines due to resonant scattering, field line wandering and small scale fluctuations in the gradient and curvature, apart from the large scale and non-diffusive drift processes. Jokipii (1966) employed the same quasi-linear theory as that used for pitch angle scattering to find

$$\frac{<\Delta x^2>}{\Delta t} = \frac{<\Delta y^2>}{\Delta t} = [\frac{\mu v}{B_o^2} P_{xx} (k=0) + \frac{(1-\mu^2) v}{2/\mu/B_o^2} P_{zz} (k = \frac{\omega_b}{\mu v})] \quad (41)$$

for $<B> \parallel z$. The second term expresses a resonance scattering between longitudinal waves and the circulating particle motion. As elaborated by Jokipii and Parker (1969) the first term is similar to that obtained for the rate of separation of flux tubes in the expanding turbulent solar wind and is therefore identified as diffusion due to the random walk of these flux tubes. This first term depends on the power in transverse waves at zero frequency and dominates over the second. A re-evaluation of its magnitude by Forman et al (1974) yields $K_\perp \cong 4 \times 10^{20} \beta \text{ cm}^2 \text{ s}^{-1}$. The second or gyro-resonance term corresponds to the $K_\perp$ values on the diagonal elements of the diffusion tensor (11) in relation to $K_\parallel$, but if field line wandering is included, these diagonal elements have to be amended.

Moussas et al 1982c have performed numerical experiments on $K_\perp$ in a manner similar to that described in 4.1, but specifically exclude the effects of field line wandering. They find $K_\perp = 8.10^{20}$ $\text{cm}^2 \text{ s}^{-1}$ at 100 MeV and $K_\perp = 2.10^{19} \text{ cm}^2 \text{ s}^{-1}$ at 0.1 MeV rather similar in magnitude to the Forman et al value. Because the cyclotron effect is small, these authors deduce that random fluctuations in small scale IMF gradients and curvature cause random drifts which add to give a $K_\perp$ contribution roughly equal to that of wandering field lines.

A review of the accumulated experimental evidence on perpendicular diffusion may be found in Palmer (1982). Important lines of evidence are the longitudinal spreading of the electron stream originating from Jupiter, the measured misalignment between the streaming anisotropy in a proton event and $\hat{B}$ and the long-term preservation of the longitudinal profiles of corotating particle events. Palmer finds $K_\perp^r \approx 10^{21} \beta \text{ cm}^2 \text{ s}^{-1}$, in reasonable agreement with the above theoretical estimates. As pointed out by Moussas et al (1982c), such values of $K_\perp^r$ mean that perpendicular diffusion dominates the total diffusion coefficient in the radial direction, $K_{rr} = K_\parallel \cos^2 \psi + K_\perp \sin^2 \psi$, at large radial distances ($\gtrsim$ 5AU).

4.3 Drift Motion in Smooth and Turbulent Fields

Until recently, the antisymmetric part of the diffusion tensor (11) has been neglected in the solution of the Fokker-Planck equation (20) or (22). This was because it was argued that the associated streaming depended upon the sign of $\hat{B}$ and in the solar equatorial regions it was clear that sector structure reversals would tend to cancel large scale effects due to this term. However the recognition that the secotr structure seems to disappear above 15°-30° solar latitude means that streaming perpendicular to the equatorial plane can be important. Jokipii et al (1977) following Levy (1975) and Fisk (1976) consider an additional divergence added to the left hand side of the

Fokker-Planck (20)

$$\frac{\partial}{\partial x_i}\left(- K^A_{ij} \frac{\partial U}{\partial x_j}\right) = \frac{\partial}{\partial x_i} <v_{iD}> U \tag{42}$$

where $<v_{iD}>$ is the i^{th} component of drift velocity due to the anisotropic part of (11) with $K^A_{ij} = -K^A_{ji}$. Taking the divergence immediately shows

$$\frac{\partial K^A_{ij}}{\partial x_j} = - <v_{iD}> \tag{43}$$

because $\partial/\partial x_i <v_{iD}> = 0$. This last statement arises from Liouville's theorem which implies that a magnetic field with a uniform particle distribution cannot create an anisotropy. (43) is equivalent to the standard drift formula obtained by averaging single particle motion over pitch angle in the guiding centre approximation as given for example by Rossi and Olbert (1970)

$$<v_\perp^G> = \frac{vp}{eB}\left\{ \frac{1}{3}\frac{\underline{B} \times \nabla \underline{B}}{B^2} + \frac{1}{3}\frac{\underline{B} \times [(\underline{B}.\nabla) \underline{B}]}{B^3} \right\} \tag{44}$$

$$<v_{\shortparallel}^G> = \frac{1}{3}\frac{vp}{eB} = (\text{curl } \underline{B})_{\shortparallel} \tag{45}$$

where the right hand side terms of (44) represent respectively the familiar field gradient and curvature drifts. (45) depends on a component of curl B parallel to the field. Since there is such a component of curl in the IMF, drift motion parallel to $\hat{\underline{B}}$ is also important. Illustrative computations by Jokipii et al (1977) show that $|<v_D>| \simeq 10^8 P \text{cm s}^{-1}$ at a rigidity P GV in the 0 → 60° solar latitude range and between 1 AU and 3 AU. These same authors provide an expression for the drift along a neutral sheet.

Because the drift velocity may exceed the solar wind velocity and thus become a dominating term in the Fokker-Planck, it is important to know whether this guiding centre approach is valid in the real, rather turbulent IMF. Isenberg and Jokipii (1979) present a quasi-linear theory based demonstration in the weak scattering limit that the guiding centre equations (44) and (45) apply, irrespective of the ratio of gyro-radius r_c to magnetic fluctuation scale size, L. They argue that this is possible even though guiding centre theory demands $r_c << L$ because on the one hand, in guiding centre theory, each particle follows a nearly helical orbit. On the other hand, in the fluctuating field, each particle follows a short section of a helix, but after a perturbation another particle takes over and follows the next section of the orbit. In this way, there is always some particle following a particular, nearly helical orbit. Lee and Fisk (1981) criticise this quasi-linear approach and put forward the example of a twisted IMF field configuration, perhaps related to solar convective cells, in which (43) does not hold. Recent numerical investigations of drift by Moussas et al (1982c) using a field model derived from IMF data showed that guiding centre theory correctly predicts drift at 100 MeV if the actual, measured values of the local gradient and curvature are used. The numerical experiment was done with data taken at 5 AU and only field fluctuations in the plane containing the sun-spacecraft line and corotation vector were retained. It was the local curvature, rather than the Archimedean pattern which dominated the drift term, but nevertheless, these results give great encouragement to the use of the guiding centre expression in the IMF situation.

4.4 Statistical Acceleration Coefficient

In this section we concentrate on the diffusion coefficient in energy space which applies generally, throughout interplanetary space. Special acceleration processes which may be associated with interplanetary shocks (e.g. Van Allen and Ness, 1967, Armstrong et al 1977) are really relevant only in the context of solar proton propagation and the production of spikes in corotating stream events and we shall neglect these processes from the viewpoint of modulation theory.

Since the original work by Fermi (1949) on cosmic ray acceleration in the interstellar medium which was formulated in terms of the collision of a charged particle with an approaching magnetised cloud, statistical acceleration has been widely considered (Davis, 1956; Sturrock, 1966; Jokipii, 1971b; Wibberenz and Beuermann, 1971; Tverskoi, 1967; Kulsrud and Ferrai, 1971; Fisk, 1976b; Hall and Sturrock, 1967; Lee and Fisk, 1980; and Achterberg, 1981). Tverskoi's work in 1967 concerned the acceleration possible in the solar wind when Alfvén turbulence is excited. He distinguished between the adiabatic scattering of particles by long wavelength fluctuations in the magnetic field which he termed 'Fermi Acceleration' and the cyclotron resonance scattering which occurs when the size of the cyclotron radius is of the order of scale size of the fluctuation. His results for a situation where the spectrum of $\underline{k}$ vectors for the waves varied as k^{-2} yielded a distribution of energetic particles which was an exponential function of energy in the asymptotic form. Jokipii (1971) also examined cyclotron resonance scattering and calculated what he termed as the 'Fermi' acceleration. Hasselmann and Wibberenz (1968) provided a more rigorous, quasi-linear theoretical treatment of the cyclotron resonance effect. Fisk (1976a) investigated the requirements of this process, based upon $D_{TT} \approx V_A^2\, T^2/K_{\parallel}$ where V_A = Alfvén speed.

To explain the observed factor of 10 increase in the corotating event proton intensity between 1 AU and 3 AU (Van Hollebeke et al, 1978) it was found that a value of $D_{TT} \simeq 1.4 \times 10^{-6}\ T^{3/2}\ \mathrm{MeV}^2\ \mathrm{s}^{-1}$ was required, implying $K_{\parallel} \simeq 1.8 \times 10^{19}\ \mathrm{cm}^2\ \mathrm{s}^{-1}$ or $\lambda_{\parallel} \sim 3 \times 10^{-3}$ AU. This small value of $\lambda_{\parallel}$ is clearly an order of magnitude less than any other estimate, based upon solar proton profile or magnetic field turbulence data. Fisk (1976b) performed a detailed, quasi-linear computation of the long wavelength, $\delta|\underline{B}|$ effect on particle energies which he termed 'transit time damping' and obtained a more satisfactory acceleration rate.

We shall distinguish now more formally the two types of acceleration discussed above and provide an approximate treatment of both acceleration coefficients. The general resonance condition for a wave-particle interaction when the wave is a weak perturbation to a uniform, static field is

$$k_{\parallel}\, v_{\parallel} - \omega + n\, \omega_b = 0 \qquad (46)$$

where the first term represents the spatial variation of the wave phase as the particle runs along the field, the second term represents the phase change with wave angular frequency ω and the third term represents the cyclotron rotation of the particle with n as an integer. The main cyclotron resonance occurs with n = 1 and ω small compared with the other terms. The change in energy may be computed by considering a particle moving at an angle ϕ to the wave motion with total energy $E = \gamma m_o c^2$, velocity $\beta = v/c$ and wave velocity $B = V_A/c$. Interactions with a series of waves takes place so that resonant pitch angle scattering changes the direction of the particle from $\cos \alpha$ to $\cos \alpha'$, measured in the moving wave reference frame. This takes place over a length $\lambda_{\parallel}$. From the relativistic transformations of energy and momentum :

$$p^* \cos \alpha = \gamma \left[-p \cos \phi + B c \frac{E}{c^2}\right]$$

$$E^* = \gamma \left[E + B c p \cos \phi\right]$$

where * means wave frame and remembering E^* = constant and $p^* \cos \alpha$ becomes $p^* \cos \alpha'$ in the moving frame, we find

$$\Delta E = \gamma^2 E \left[1 - \frac{1}{\gamma^2} + B\beta \cos \phi \left(1 - \frac{\cos \alpha'}{\cos \alpha}\right) - B^2 \frac{\cos \alpha'}{\cos \alpha}\right] \quad (47)$$

In the non-relativistic limit with $\underline{k} \parallel \hat{\underline{B}}$, $\phi = 45°$, $\alpha = 45°$, $\alpha' = 135°$,

$$\Delta E \approx \sqrt{2}\, E\, v \frac{V_A}{c^2} \qquad \text{and with } \Delta\tau = \frac{\lambda_{\parallel}}{v_{\parallel}},$$

$$D_{TT} = \frac{|\Delta E|^2}{2\Delta\tau} = \frac{2}{3} T^2 \frac{V_A{}^2}{K_{\parallel}} \quad (48)$$

similar to the equation used by Fisk (1976a) and others for the cyclotron effect.

We now turn to the acceleration due to the long wavelength magnetosonic mode, termed transit line damping (Fisk 1976b) or small amplitude Fermi acceleration (Achterberg 1981) and we use elements from the presentation of these two authors in the following physical approach. Start from the resonance condition with n = 0 (Cerenkov resonance) in (46),

$$k_{\parallel} v_{\parallel} = \omega \quad (49)$$

Since for energetic particles $v_{\parallel} > u$ (which is the phase speed of the wave) and also $\omega = \pm uk$ is a reasonable approximation to the dispersion relation in the IMF for the magnetosonic mode, we must have $k_{\parallel} \ll k$ or $k_{\parallel} \ll k_{\perp}$ and

$$k_{\parallel} \approx \pm \frac{u}{v_{\parallel}} k_{\perp} \quad (50)$$

Thus the acceleration can only be due to waves propagating at large angles to $\hat{\underline{B}}$ and the wavelength is also long compared with the particle gyroradius.

It is instructive to employ a pair of equations (51a and 51b) documented for example by Sivakhin (1965) which we derive as follows :

Integrate the equation $\nabla.\underline{B} = 0$ in cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}(r\,B_r) + \frac{\partial B_z}{\partial z} = 0 \qquad \text{or} \qquad B_r = -\frac{1}{2}\,r\,\frac{\partial B}{\partial z}$$

for z parallel to $\hat{B}$. Set r equal to the gyroradius of a particle moving in this field which we take to be slowly converging field lines, i.e. a compression wave. The parallel component of the Lorentz force is

$$m\frac{dv_{\parallel}}{dt} = \frac{e}{c}\,v_{\perp}\,B_r \qquad \text{or} \quad m\frac{dv_{\parallel}}{dt} = -\,m\,\frac{v_{\perp}\,v_{\perp}}{2B}\,\frac{\partial B}{\partial z}$$

$$\text{or} \qquad \frac{dp_{\parallel}}{dt} = \frac{-p_{\perp}}{2B}\,v_{\perp}\,\frac{\partial B}{\partial z} \tag{51a}$$

Varying parallel electric fields are neglected on the grounds that we are interested in hydromagnetic waves where $\underline{B}.\underline{E} = 0$. However for $dp_{\perp}/dt$, the contribution of $(\text{curl } E)_{\parallel}$ is important and E_{ϕ} directed around the particles' orbit at $r = r_c$, the gyroradius, is given by $2\pi\ r_c\ E/\pi\ r_c^2 = -1/c\ \partial B/\partial t$, from which one finds an electric field contribution to $dp_{\perp}/dt$ of

$$\left(\frac{dp_{\perp}}{dt}\right)_E = \frac{p_{\perp}}{2B}\left(\frac{\partial B}{\partial t}\right)$$

The main contribution of magnetic fluctuations to $\nabla\, v_{\perp}$ arises from $(-v_{\parallel}\ B_r)$ in the cross product $\underline{v} \times \underline{B}$, hence

$$m\left(\frac{dv_{\perp}}{dt}\right)_B \approx \frac{me}{c}\,v_{\parallel}\,\frac{r_c}{2}\,\frac{\partial B}{\partial z} \quad \text{and finally}$$

$$\frac{dp_{\perp}}{dt} = \frac{p_{\perp}}{2B}\left(\frac{\partial B}{\partial t} + v_{\parallel}\,\frac{\partial B}{\partial z}\right) \tag{51b}$$

For the n = 0 resonance, equation 49, an immediate consequence of (51b) is that $p_{\perp}$ = constant during the wave particle interaction. Hence only $p_{\parallel}$ can change in the rest frame and the first adiabatic invariant is <u>not</u> conserved in the rest frame.

We relate wavenumber and gradient by $k_{\parallel} = 1/\lambda_z$, $1/B\ (\partial B/\partial z)_{\text{resonance}} = \eta\ k_{\parallel}/2\pi$ where $\eta^2 = P(k_{\parallel})\ dk_{\parallel}$, $P(k_{\parallel})$ being the fractional power at the Cerenkov resonance and $dk_{\parallel}$ the resonance width. According to the approximate dispersion relation (49), all long wavelength waves with wave vectors at the correct angle given by $\cos^{-1}\ (k_{\parallel}/k) = \theta$ contribute to the scattering, so $dk_{\parallel}$ measures the angular spread of the waves which can resonate. From (51a), the acceleration then becomes

$$\left(\frac{dp}{p}\right)^2\frac{1}{dt^2} = \frac{v_{\perp}^{\,4}}{4v^2}\,\eta^2\,\frac{k_{\parallel}^{\,2}}{4\pi^2}$$

The problem now arises as to what fraction of the observed power in $\delta|\underline{B}|$

fluctuations or fast mode HM waves is actually moving in the correct direction. One is tempted to assume an isotropic $\underline{k}$ vector distribution and write $P(k_{\parallel})\, dk_{\parallel} = (P(k)/4\pi)\, 2\pi \sin\theta\, d\theta\, dk$. Then relating the fluctuation frequency ν as measured by satellite magnetometer to k via $\nu = V\, k/2\pi$, allowing all wave numbers up to the resonance condition to take part in the acceleration ($k_{max} = 2\pi/r_c$) and using $\Delta\tau = \lambda_{\parallel}/v_{\parallel}$ for the interaction time, we eventually get

$$\frac{4D_{pp}}{p^2} = \frac{D_{TT}}{T^2} = \frac{v_{\perp}^{4}}{v_{\parallel}^{4}} \frac{V_A^3}{8v^2} \frac{\lambda_{\parallel}}{V^2} \nu_{max}^{\;3-n} \frac{P_o}{(3-n)} \tag{52}$$

where $P(\nu) = P_o\, \nu^{-n}$ and $D_{TT} = v^2\, D_{pp}$ (Fisk 1976a). At $v_{\perp} = v_{\parallel}$, we find $D_{TT}/T \approx 4 \times 10^{-8}$ MeV s^{-1}, rather smaller than the Fisk (1976b) estimate.

Moussas et al (1982a) have adapted their numerical methods to include the effect of the varying interplanetary electric field. They follow particle trajectories in a field model derived both from magnetometer data and from $\underline{E} = -\underline{V} \times \underline{B}$ where $\underline{V}$ is obtained from spacecraft plasma flow measurements. The change of energy can be found at the end of each 'layer' of magnetic field. Following the method employed for spatial diffusion, particles are injected into the model at one energy and removed at boundaries located at a higher and a lower energy. The mean drift in energy space and the slope of the steady state distribution function determined in the numerical experiment yield D_T and D_{TT} (see (28)). Because of equations (28), (29), these coefficients are related by $D_T = \partial/\partial T\, D_{TT} + D_{TT}/2T$ and it is a useful check on the numerical integrations that in fact the D_T and D_{TT} values calculated do obey this equation, which is a consequence of Liouville's theorem. The final result is that $D_{TT} \approx 4 \times 10^{-6}\, T^{1.5}$ MeV^2 s^{-1} at 5 AU and perhaps a factor 3 to 4 higher at 1 AU.

Taking into account $K_{\parallel} \propto T^{1/2}$ at low energies, equation (48) for the theoretical cyclotron resonance Fermi acceleration satisfies $D_{TT} \propto T^{1.5}$ which is a power law that fits the computational result. However in absolute magnitude, the numerically computed D_{TT} is one or two orders of magnitude larger than any of the theoretical estimates. A way around this difficulty may be in the fact that the Fermi mechanism is most effective when the scattering is at large pitch angles. From (47) one can show that if $\phi \sim \alpha \sim \alpha' \sim \pi/2$, $\Delta E \approx EB_{\beta}\, \delta\alpha$ for a small scatter, $\delta\alpha$. However, at $\phi \sim \alpha \sim \alpha' \sim 0°$, a similar small scattering yields $\Delta E \approx EB_{\beta}\, \delta\alpha^2/2$. It may be that repeated scattering at $\alpha \to \pi/2$ in one relaxation time $\Delta\tau \sim \lambda_{\parallel}/v_{\parallel}$ greatly enhances the acceleration efficiencies over that obtained by a simple averaging procedure such as led to (48).

Moussas et al (1982a) discuss a wide variety of experimental data which can be used in favour of statistical acceleration rather than shock acceleration as the origin of co-rotating stream events, thus supporting the idea that the acceleration coefficient is large.

5. Approximate Solution to the Modulation Equation

Various approximate ways have been presented for the solution of the modulation transport or Fokker Planck equation (22), under steady state conditions. The emphasis is often to give physical insight into the modulation process. We have already given one in the introductory section 1, involving a constant Compton-Getting factor. Another is the

'Force-Field' solution of Axford and Gleeson (1968).

5.1 The Force-Field Solution

Based on the experimental result that radial streaming is small, a version of (21) written in terms of the isotropic part of the distribution function, f_o, can be used :

$$S_r = -4\pi\, p^2 \left\{ \frac{V_p}{3} \frac{\partial f_o}{\partial p} + K_{rr} \frac{\partial f_o}{\partial r} \right\} = 0 \qquad (53)$$

Note that the total distribution function $f(\underline{r},\underline{p})$ has an anisotropic part $f_1 \sim V/v\, f_o$ (cf. Gleeson 1969) and that (53) comes from (21) written in terms of momentum with U transfored to f_o and employing (16).

Mathematically, (53) states that f_o is constant along lines in $\underline{r}$, $\underline{p}$ space defined by the characteristic equation

$$\frac{dp}{dr} = \frac{pV}{3K_{rr}} (r,p) \qquad (54)$$

(53) is also equivalent to the Liouville equation in a conservative field with a "force" $pV_r/3K_{rr}$. A group of particles entering the solar modulation cavity follow contours of constant f_o in the r-p plane and as shown by Fisk, Forman and Axford (1973) they suffer adiabatic deceleration all the time and after reaching a minimum value of r, they turn around and are finally convected back to the boundary at a much reduced energy. This process will be discussed in greater detail in section 6. We note however with Fisk et al that at the minimum r value, $df_o/dp = 0$ and hence $C = 0$, corresponding to obverations in the 30-200 MeV region that $C \approx 0$.

An interesting and related solution occurs if the diffusion coefficient is a separable function of r and p, i.e. $K_{rr}(r,p) = K_1(r)\, K_2(p)\, \beta$. It is not obvious from (33) that this is necessarily so, but in practice at solar proton energies, $K_1 \approx$ constant and $K_2 \approx$ constant. Little is known about K_1 at neutron monitor energies. Integration of (54) yields

$$\int_p^{p_\infty} \frac{K_2(p')\beta(p')}{p'}\, dp' = \int_r^{r_\infty} \frac{V}{3K_1(r')} dr' \equiv \phi(r) \qquad (55)$$

where p_∞ is the entry momentum at r_∞. $\phi(r)$ is the modulation parameter introduced by GLeeson and Axford (1968b). If we consider the higher rigidity range $R \gtrsim 0.1$ GV where $R = pc/ze$ and where $K_2 \approx R$ and is conventionally measured in units of magnetic rigidity, the left hand side of (55) becomes

$$\phi(r) = \int_p^{p_\infty} \frac{p'}{p'} \frac{C\beta}{ze} (p')\, dp' = \frac{1}{2} \frac{1}{e} \frac{\{(E_\infty)^2 - E^2\}}{Z\, M_\infty}$$

If $E_\infty \approx E$ where $E^2 = P^2 + M_o^2$(units of eV^2), i.e. the modulation is small,

$$\phi(r) \cong \frac{\Delta E}{Ze} \tag{56}$$

Now in terms of energy, conservation of f_o and Liouville's theorem imply

$$\frac{j_E(r,E)}{E^2-E_o^2} = \frac{j_E(r_\infty,E\ )}{E_\infty^2-E_o^2} \tag{57}$$

where j_E is mean differential intensity with respect to energy. Under the approximation leading to (56) and (57), the intensity is therefore given by

$$\frac{j_E(r,E)}{E^2-E_o^2} = \frac{j_E\ (r_\infty,E_\infty)}{(E+Ze\phi)^2-E_o^2} \tag{58}$$

This result is equivalent to that given by Ehmert (1960) for positively charged particles moving under the influence of a heliocentric electric field $E(\underline{r},t) = V/3K_1(r_1)$ without an electric potential $\phi(r)$.

Gleeson and Urch (1973) discuss the validity of this force-field approach in relation to the full numerical solution of the modulation equation and its breakdown, dependent on the nuclear species involved, in the region below about 200 MeV. Notice that with ϕ in units of GV, an estimate of the adiabatic energy loss is provided by $\Phi = Ze\phi$ in GeV, subject to restricting this result to relatively high energies. Also note the relationship of ϕ to M in (3). For C $\sim$ 0.8 as observed at 1 GV, M $\sim$ 2.4 ϕ.

5.2 Energy Loss by Drift - Kota Process

It was in 1965, following a suggestion by Dungey, that Houghton (1965) calculated the drift of a solar proton under gradient and curvature drift in the IMF and pointed out that this drift was always in a direction such that particles lost energy against the $\underline{E} = -\underline{V} \times \underline{B}$ field, irrespective of the sign of $\underline{B}$. Kota (1979) basically uses this drift-energy loss effect as an approximation to compute the amount of modulation. Kota is concerned to explain the solar modulation cycle by long term changes in the latitude extension of the sector structure - an idea we will mention later in connection with the work of Jokipii and Thomas (1981) - and it takes its inspiration from an earlier paper, Erdős and Kota (1979). However the physical idea is of most interest at this point. Barnden and Berkobitch (1975) had the same general idea.

Kota (1979) starts by deriving the energy loss due to drift,

$$\left(\frac{dE}{dt}\right)_d = -\frac{pv}{3}\,\text{div}\ (V \sin^2 \psi) \tag{59}$$

and that due to scattering

$$\left(\frac{dE}{dt}\right)_s = -\frac{pv}{3}\,\text{div}\ (V \cos^2 \psi) \tag{60}$$

These results seem related to those obtained by Webb and Gleeson (1979), see equation (24) and (25), if we make the reasonable assumption that transforming the betatron effect from the moving to the fixed frame yields the drift effect and transforming the Inverse Fermi effect from the moving to the stationary frame yields the scattering effect (see also Webb et al (1981). Because most modulation takes place in regions where $\tan \psi > 1$, ψ being the angle between $\hat{\underline{r}}$ and $\hat{\underline{B}}$, Kota deduces that the drift loss term dominates in the contribution to the total adiabatic loss rate, $dE/dt = -pv/3 \text{ div } V$. If we neglect scattering and represent the $\underline{E} = -\underline{V} \times \underline{B}$ solar wind electric field by a potential, then $\Phi_N = -\Phi_s$ where

$$\begin{aligned} \Phi_N &= 150 \sin \lambda \text{ MV, pre-1969 field reversal} \\ \Phi_N &= -150 \sin \lambda \text{ MV, post-1969 field reversal} \end{aligned} \tag{61}$$

for solar latitude λ and we have used equation (4). Equation (7) represents the neutral sheet boundary between Φ_N and Φ_S. There is a mathematical discontinuity in Φ at the wavy neutral sheet boundary. It does however mean that finding the total energy loss reduces to adding up the potential jumps at the neutral sheet crossings. This is because

$$\Delta E = Ze \int \underline{E}.\underline{ds} = Ze \{ \phi_B - \phi_G + \sum_{i} \phi_1 - \phi_H + \phi_3 - \phi_2 + \phi_5 - \phi_4 \ldots \} \tag{62}$$

where ϕ_G is the potential in the distant galaxy, ϕ_H the potential after crossing the heliosphere boundary, ϕ_1 and ϕ_2 the potentials either side of the first neutral sheet crossing, ϕ_3 and ϕ_4 the potentials either side of the next neutral sheet crossing, etc., and ϕ_B is an additional boundary potential, depending on the details of curl $\underline{E}$ at the boundary (Erdös and Kota 1978). But $\phi_2 = -\phi_1$, $\phi_4 = -\phi_3$, etc. so

$$\Delta E \approx Ze \, (2\phi_1 + 2\phi_3 \ldots)$$

for multiple crossings. Thus the amount of energy loss, or depth of modulation, can be related to the number of neutral sheet crossings for particles which move especially in the solar equatorial plane, due to some scattering diffusion or (curl B)„ drift. For example, in the post 1969 solar cycle, protons drift in from polar regions suffering a large energy loss under (61) while the equatorial plane motion of electrons where the tendency is to migrate to higher solar latitudes suggests much less modulation. However, if electrons can experience multiple crossings of the wavy neutral sheet, their depth of modulation is enhanced and the difference in modulation between the two species is not so great (Kota 19790. Clearly the waviness of latitude extent of the sector structure influences the depth of modulation and a solar cycle variation of this could be the unknown factor in causing the 11- year wave.

6. Steady-state Monoenergetic Source Solutions

Early analytical solutions to the Fokker-Planck equation for special cases including the spiral geometry but with no drift or $K_\perp$ were given by Parker (1965, 1966) and a number of other special solutions are available in the literature (Fisk and Axford, 1969; Cowsick and Lee, 1977; Lee 1976; Gross, Lee and Lerche 1977;

Dolginov and Toptygin 1967, 1968; Webb and Gleeson, 1976, 1977). However use of Green's Function methods by Webb and Gleeson (1973) and Toptygin (1973) enables more insight to be achieved concerning the flow lines in $\underline{r}$ - $\underline{p}$ space and also on the contribution of particles at various energies on the boundary of the heliosphere to the near earth spectrum.

The Green's Functions used are limited to spherically symmetric models of modulation with a constant solar wind speed, V, and the diffusion coefficient $K = K_o(p)\, r^b$ where $K_o(p)$ is an arbitrary function of momentum. (27), the momentum form of the Fokker Planck, is then solved with $\underline{\underline{K}}.\partial U_p/\partial r = K_r\, \partial U_p/\partial r$ in the form

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V U_p - r^2 K_r \frac{\partial U_p}{\partial r}\right) - \frac{2}{3}\frac{V}{r}\frac{\partial}{\partial p}(p\, U_p) = N\,\frac{\delta(r-r_o)\;\delta(p-p_o)}{4\pi\, r_o^{\,2}} \qquad (62)$$

Under the approximation $r_o \rightarrow \infty$, that is the particle injection takes place at a distant boundary, a limiting solution for b > 1 was obtained by Webb and Gleeson (1973) for the mean distribution function $f_o = U_p/4\pi\, p_o^{\,2}$

$$f_o = \frac{3K_o(p_o)\; N_g}{8\pi\; p_o^{\,3(1+b)/2}\;\Gamma(m)}\;\frac{1}{\tau}\left(\frac{x^2}{4\tau}\right)^m \exp\left(-\frac{x^2}{4\tau}\right) \qquad (63)$$

where $U_p(\infty, p_o) = N_g\, \delta(p-p_o)$

$$x = \frac{2}{(1-b)}\,(rp^{3/2})^{\left(\frac{1-b}{2}\right)}$$

$$n = (b+1/(1-b)\;;\; m = |n|$$

$$\tau = \frac{3}{2V}\int_p^{p_o} K_o(z)\; z^{(1-3b)/2}\, dz$$

$$x_o = x_o(r_o, p_o)$$

Figure 7 represents this solution for b = 1.5, $K_o \propto p$ and plots $f_o/N_g\, p_o^{\,3}$ as a function of p/p_o for values of $Vr/K(r,p_o)$ equal to 1.0, 0.1, 0.01 and 0.001. If we take a value of K_o which is able to reproduce the 1965 level of modulation, the curves Vr/K = 0.01, 0.1, 1.0 represent respectively the distribution at 1 AU of T_o = 20 MeV, 1200 MeV and 20 GeV. The great spreading and appearance of secondary peaks at low energy, relative to the monoenergetic injection distribution function, is seen in Figure 7. Alternatively, Figure 7 can be interpreted as providing the radial variation at one energy. If T_o = 1200 MeV and Vr/K represents the distribution at r = 1 AU, the other curves represent the results of this release at r = 100 AU and 0.01 AU since $\frac{Vr}{K} \propto r^{-1/2}$.

The monoenergetic solution can be used to investigate the inter-relationship between the particle flow and energy transfer between the cosmic rays and the solar wind using the concept of flow lines already mentioned previously in connection with the force field solution. Fisk, Forman and Axford (1973) were able to obtain their r-p plane contours by virtue of the special case that gradient driven flow and the Compton-Getting correction term balanced, but a more general, although still spherically symmetric, case is obtained following Webb (1976). Take the conservation of flow in position and energy space

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 S_p) + \frac{\partial}{\partial p}(\langle\dot{p}\rangle U_p) = 0$$

which is (20) in terms of momentum and where

$$S_p = VU_p - \frac{V}{3}\frac{\partial}{\partial p}(p\, U_p) - K_r \frac{\partial U_p}{\partial r}$$

Thus we can allow a finite S_p by balancing it with a source in momentum space. The flow lines of Webb (1976) are solutions of the differential equations

$$\frac{dr}{dt} = \langle\dot{r}\rangle = \frac{S_p}{U_p} \tag{64a}$$

$$\frac{dp}{dt} = \langle\dot{p}\rangle = \left(\frac{p}{3U_p}\right) V \frac{\partial U_p}{\partial r} \tag{64b}$$

(64b) being equivalent to (19) in terms of momentum. The flow lines which define the average effects of the interplanetary medium on the particles are then given by

$$\frac{dr}{dp} = \frac{3S_p}{pV}\frac{\partial U_p}{\partial r} \tag{65}$$

Examples of flow lines for the case $K = K_o\, pr^{1.5}$, $Vr_e/K(r_e,p_o) = 0.1$ are shown in Figure 8. They were obtained by using the solution (63) in conjunction with (64a) and (64b). It is seen that there are two types of flow lines. Some go inward and then turn outward, always dropping monotonically in energy. Others, starting above the critical curve indicated by long and short dashes, first fall in energy on entry but then gain energy as they turn around and eventually emerge. The locus of $\langle\dot{p}\rangle = 0$ corresponds to the peaks in the distribution function of Figure 7 (dot-dash curve) while the locus of $\langle\dot{r}\rangle = 0$ separates regions of inwards and outwards flow (dashed line).

It is reassuring to find in Figure 8 a region of r-p space where particles are gaining energy. This region presumably corresponds to the average energy gain implied by (19) since $\partial U/\partial r$ is positive for galactic cosmic rays. Remembering that these paths refer to groups of particles on average, not single particles, it is not surprising that incoming cosmic rays seeing scattering centres approaching them can gain energy by a second order Fermi process. However, some groups of particles will experience more tail on collision and be decelerated. The term in the Fokker Planck (22) associated with $\partial/\partial T$ which was given the name adiabatic deceleration and tries to make $\partial U/\partial t$ negative is actually dominated by the contribution of the energy dependent part of the Compton-Getting transformation in our derivation. Thus the term really represents an enhancement of the sweeping effect of the solar wind with which the cosmic ray gas is attempting to get into equilibrium. Thought of this way we do not have the incompatibility of a term labelled 'deceleration' in the stationary frame when the effect of the wind can be a heating, as seen by (19). It is also helpful to look at (53) with $S_r \approx 0$. It is the convection of particles with $\underline{V}$, coupled with what must be a negative value of $\partial f_o/\partial p$, bringing more particles into a given momentum interval with transformation to the laboratory frame that provides the current to balance the diffusion current.

The limiting exact solution (63) can be used as a Green's Function for a monoenergetic spectrum at infinity and convoluted with the galactic spectrum to generate a full solution for the intensity anywhere within the solar cavity. Thus

$$f_o(r,p) = \int_p^\infty f_o(\infty,p_o)\, G(r,p;p_o)\, d\, p_o \qquad (66)$$

where $G(r,p;p_o)$ is obtained from (63). Indeed, the previous work of Fisk and Axford (1969) and subsequent work of Cowsick and Lee (1977) seem to both stem from this same basic solution. Gleeson and Webb (1979) compare the integral (66) with the predictions of numerical solution of the spherically symmetric Fokker-Planck as performed by Urch and Gleeson (1972) and show that very similar results are obtained, thus confirming the use of (66) as valid in other investigations.

Figure 9 represents the Urch and Gleeson numerical result, rather than (66) and shows the modulation for the three types of galactic spectrum, (a), (b) and (c) as illustrated. Near-earth spectra are calculated with $K_r = 6 \times 10^{21} \sqrt{r}\, R\, \beta$ cm^2 s^{-1} the force field parameter ϕ(1 AU) = 0.14 GV and r_∞ = 10 AU as the heliosphere boundary. It is important to note how insensitive the near-Earth spectrum is to the exact form of the low energy galactic spectrum.

It is clear from our previous discussion of (63) that the Green's function can be used to show the contribution of various momentum ranges of galactic particles to the near-Earth intensity, a point taken up by Gleeson and Webb (1975). Numerical solutions which made a similar point were carried out by Goldstein et al (1970). These workers used the same spherically symmetric Fokker-Planck for protons and helium nuclei separately (Figure 10). We see from this figure that at high energies, the galactic spectrum at a particular energy makes a large contribution to the modulated spectrum in the same energy range. At lower energies, the galactic spectrum has little effect on the modulated spectrum at the same energy and most particles actually seen near the earth have been shifted down in energy from $\gtrsim$ 100 MeV. These conclusions are model dependent. In order to match the measured cosmic ray gradients $\sim$ 1%/AU, values of $K_\parallel$ much greater than those suggested by quasi-linear theory or even the numerical calculations of Moussas et al (1982b) are employed. Gleeson and Webb (1975) for example use $K_r \sim 1.5 \times 10^{22}$ cm^2 sec at 1 GV. As we shall see later, a 3-dimensional model is required for modulation and some substantial modifications to the conclusions arising from Figures 9 and 10 may be necessary.

Webb and Gleeson (1977) develop a Green's Formula for the transport equation which is a generalisation of their previous Green's Function methods. By means of this formula, they can solve problems in which the intensity and streaming on the boundary of the modulation cavity can be expressed in terms of the solution for a source which is a delta function in the independent variables t, $\underline{r}$, $\underline{p}$. Kota's (1977) time reversed method is related to this work. The Webb and Gleeson (1977) formula gives the intensity at a point in terms of three integrals; one over the volume enclosed by the boundary and determined by the number of particles which go backwards in time to sources which are within the volume, the second given by the number of particles which reverse to match with the current through the boundary and the third to correspond to the effects of an initial source distribution. Lerche's (1974) variational method also depends upon the use of a differential Green's theorem and similar integrals. Lerche solves the transport equation by using trial functions for the differential number density and then extremising the Lagrangian operator representing the transport equation.

7. Spherically Symmetric Modulation Solutions and their Problems

7.1 The Diurnal Variation

Although not giving a complete story, some overall understanding of modulation throughout the Heliosphere is achieved by assuming no latitude or longitudinal dependent gradients, as we have seen in the previous sections 5 and 6. Also the basic observation of a 0.4% anisotropy, roughly perpendicular to the earth-sun line and coming from the East in the 2-20 GeV proton energy range, can be explained (Parker 1964, 1967; Axford (1965; Pomerantz and Duggal, 1971; Urch and Gleeson 1972). This is the dominant anisotropy, on average. We may write the streaming equation (21) more explicitly as

$$\underline{S} = CU\underline{V} - K_{\parallel}\left(\frac{\partial U}{\partial \underline{r}}\right)_{\parallel} - K_{\perp}\left(\frac{\partial U}{\partial \underline{r}}\right)_{\perp} - \frac{1}{3}\frac{v^2}{\omega_b^2}\left(\frac{\partial U}{\partial \underline{r}}\right) \times \hat{\underline{B}} \qquad (67)$$

for $\omega_b/\nu_c >> 1$ and $K_{\perp}$ including all causes of perpendicular diffusion. In spherical polars, (67) becomes

$$\xi_r = \frac{3}{v}\left(CV - K_{rr}\frac{\partial U}{\partial r}\right) \qquad (68a)$$

$$\xi_\theta = \frac{3}{v} K_T \sin\psi \frac{\partial U}{\partial r} \qquad (68b)$$

$$\xi_\phi = \frac{3}{v}(K_{\parallel} - K_{\perp}) \sin\psi \cos\psi \frac{\partial U}{\partial r} \qquad (68c)$$

where $\underline{\xi} = 3\underline{S}/vU$, $K_T = v^2/3\omega_b$. For zero radial flow (54) yields

$$\xi_\phi = \frac{3C\ v/V\ (K_\parallel - K_\perp)\ \sin\psi\ \cos\psi}{K_\parallel \cos^2\psi + K_\perp \sin^2\psi} \qquad (69)$$

If $K_\parallel >> K_\perp$; $\xi_\phi = 3C\ v/V \tan\psi = 3C\,\Omega\, r \sin\theta\ /V$ corresponding to the perfect corotation of the cosmic ray flux with the Archimedes spiral field pattern at rotational speed Ω which is that of the sun. For T > 1 GeV, C = 1.5 and with V = 400 km s^{-1}, $\theta = \pi/2$ we find $\xi_\phi = 0.6\%$. Subramaniam (1971) discusses small but cumulative errors which together may reduce the calculated value of ξ_ϕ to that observed.

Although the definitive work by McCracken and Rao (1965) on the 1958-1964 neutron monitor data gave a phase angle for the diurnal anisotropy of 86.5 ± 1.6° east of the Earth-Sun line, many analyses of experimental results have drawn attention to departures from this near perfect corotation. For example, Thambyahpillai and Elliot (1953) suggested that there is a 22-year wave in the anisotropy and more recently, Forbush and Beach (1975) analysed data in terms of a corotating component plus a 20 year wave with a maximum at 128°E, perhaps corresponding to flow into or outward from the sun along the Archimedes spiral field direction.

Swinson (1971) has drawn attention to the information on the radial gradient which may be obtained from a measurement of the north-south anisotropy, ξ_θ. The sign of the effect is correlated with that of the sector structure, as can be seen via the last term of (67). Both diurnal variation data and north-south anisotropy in the absolute cosmic ray flux as seen by polar neutron monitors have been interpreted in this way (e.g. Pomerantz, Tolba, Duggal, Tsao and Owens, 1981). Swinson found a radial gradient at high rigidities which, when extrapolated to 1 GV yielded about 14%/AU or more.

7.2 Numerical Solutions

In order to accommodate a wide range of parameters describing the position and rigidity dependence of the primary spectrum numerical methods have been developed to solve the spherically symmetric Fokker-Planck transport equation (Fisk, 1969; Urch 1971). The Crank-Nicholson technique is often used. A galactic spectrum is specified at the boundary of modulation $r = r_\infty$, and the partial differential equation, $U = U(r,T)$ integrated inwards in r between a low energy cutoff where $U(T)$ is zero and a high energy cutoff where the spectrum remains at the galactic value. The rigidity dependence of the diffusion coefficient can be obtained empirically by assuming the electron modulation is completely specified by the near-earth measured spectrum and the galactic spectrum as deduced from the non-thermal radio background (Goldstein et al, 1970; Burger, 1971). There are of course many models for the galactic proton intensity which show large variation with position in the galaxy and therefore the assumption of the existence of an average galactic electron spectrum which is applicable to this "demodulation" procedure cannot be too safe. Nevertheless, values of the total modulation parameter $M \cong 2.4\ \phi$ (equation 3) have been deduced, for example ϕ = 0.35 GV in 1965 (Urch and Gleeson, 1972), ϕ = 0.59 GV (Bedijn et al, 1973), again in 1965, and ϕ = 0.44 GV for 1973 (Garcia-Munoz et al, 1977). Checks on the modulation model can then be made by predicting the galactic proton and helium spectra at a particular epoch of the solar cycle. Next, these galactic spectra are used to predict new, near earth spectra at another epoch.

Although some satisfactory fits to observed spectra have been obtained by the numerical models, it is not possible to disentangle the absolute magnitude of $K_{\parallel}$, the radial dependence of $K_{\parallel}$ and the position r_{∞} which all appear in M. In fact, early estimates of $K_{\parallel}$ were later revised upwards to fit the additional information given by the low $\sim$ few %/AU gradient measurements made on Pioneers 10 and 11 (Gleeson and Webb, 1979). A recent value for this gradient is provided by Webber and Lockwood 1981 for $T > 60$ MeV averaged out to 23 AU. These workers find 2.85 ± 0.5%/AU and incidentally deduce $r_{\infty} >$ 65 AU from the dependence of the gradient on counting rate based on spherically symmetric modulation theory. A value $\lambda_r = 0.3$ AU is consistent with their work.

Three basic problems arise in the application of spherically symmetric solutions. The first lies in the value of $K_{\parallel}$ adopted which we have already seen is considerably higher than the values at 1 and 5 AU which are given by the latest numerical simulations. Second, there is no simple explanation of the hysteresis shown by the lag in the recovery of the electron intensity relative to the protons, seen after the 1970 sunspot maximum (Rockstroh, 1977). Third there is no known variation in the near Earth solar wind velocity, or power spectrum of field irregularities to explain the required variation of M (Mathews et al 1971; Hedgecock et al, 1972).

Morfill et al (1979) have provided a possible way out of the third difficulty in the context of the spherically symmetric model. It depends on the observation (Hedgecock 1975) that the power density in low frequency waves ($< 10^{-4}$ Hz) does vary with the solar cycle, unlike the other IMF parameters. If the $\underline{k}$ vectors of the Alfvenic fluctuations align with the radial direction, the fact that the mean local field direction alters with the solar cycle due to the change in the low frequency power means that the scattering can change. However there remains the problem of demonstrating the radial alignment of $\underline{k}$.

8. Three-dimensional Modulation - Perpendicular gradient and Anisotropy Evidence

In the previous section we have already mentioned the problems in spherically symmetric modulation theory that lie with predicting the low radial gradient, accounting for positive-negative particle hysteresis effects and finding the actual cause of the 11-year cycle. In section (5.2), the energy loss formulation of Kota was essentially a 3-D model and therefore it is reasonable to search for other evidence to support the need for a full 3-dimensional development of the complete theory.

Perpendicular gradients and related anisotropies are direct manifestations of a non-spherically-symmetric cosmic ray distribution. Before discussing the rather confusing evidence for these, it is helpful to develop equation (67) to take into account perpendicular and azimuthal gradients in order to see the gradiant-anisotropy inter-relationship. In component form, with $S_i = v/3\ \varepsilon_i$, (67) becomes

$$S_r = CUV - K_{rr}\frac{\partial U}{\partial r} + K_T \sin\psi \frac{\partial U}{\partial p} + (K_{\parallel}-K_{\perp}) \sin\psi \cos\psi \frac{\partial U}{\partial a} \quad (70a)$$

$$S_p = -K_T \sin\psi \frac{\partial U}{\partial r} - K_\perp \frac{\partial U}{\partial p} - K_T \cos\psi \frac{\partial U}{\partial a} \quad (70b)$$

$$S_a = (K_\parallel - K_\perp) \sin\psi \cos\psi \frac{\partial U}{\partial r} + K_T \cos\psi \frac{\partial U}{\partial p} - K_{aa} \frac{\partial U}{\partial a} \quad (70c)$$

where $K_{rr} = K_\parallel \cos^2\psi + K_\perp \sin^2\psi$; $K_{aa} = K_\parallel \sin^2\psi + K_\perp \cos^2\psi$; $K_T = v^2/3\omega_b$ with $\hat{\underline{p}}$ a unit vector in the θ or north → south direction and $\hat{\underline{a}}$ a unit vector east to west or in the ϕ direction.

Neglect $\partial U/\partial a$ as being due to transient, azimuthal effects which are likely to average out in the long-term. (70b) and (70c) then combine to yield

$$\langle S_a \rangle = (K_\parallel - K_\perp - \frac{K_T^2}{K_\perp}) \sin\psi \cos\psi \frac{\partial U}{\partial r} - \frac{\langle K_T S_p \rangle}{K_\perp} \cos\psi \quad (71)$$

If there is no perpendicular gradient, $\partial U/\partial p = 0$, $\langle K_T S_p \rangle = K_T^2 \sin\psi\, \partial U/\partial r$ from (70b). It is clear that S_p and K_T switch sign together with sector structure reversal and (71) reduces to (68c). It is interesting also to note that although the sum $K_\parallel - K_\perp - K_T^2/K_\perp$ in (71) reduces to zero if $K_\perp = K_\parallel \nu_c^2/\nu_c^2+\omega_b^2$ as in (11) and in cyclotron resonance theory for $K_\perp$ (McDonald and Forman 1981b), the north-south streaming due to (70b) coming in the term $\langle K_T S_p \rangle/K_\perp$ results in there still being an azimuthal diurnal variation.

The first order equation (70) cannot deal with symmetric perpendicular gradient effects, for example the situation where the helio-equator represents a minimum or maximum in intensity. Apart from full numerical integration of the Fokker-Planck, approximate insight into the magnitude of the streaming which can result can be obtained following the work of Jokipii and Parker (1968) and Quenby and Hashim (1969). These last authors write the Fokker-Planck, retaining "θ" component non-symmetric terms in $K_\perp$ as

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2\, CUV) = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 K_\perp \frac{\partial U}{\partial r} + \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} K_\perp \frac{\sin\theta}{r} \frac{\partial U}{\partial \theta}$$

$$- \frac{1}{r^2} \frac{\partial}{\partial r} r^2 K_\parallel \frac{\partial U}{\partial r} \sin^2\psi \quad (72)$$

If $U = U_1 + U_2$ for $U_2 << U_1$ where U_1 is an analytical solution of the modulation equation with $K_\perp \equiv 0$ and $K_\parallel$ a simple power law in r, as found by Parker (1965), the term in $K_\perp$ equal to $K_\perp/r^2\, \partial^2 U/\partial\theta^2$ is the dominating non-symmetric term. Then the radial streaming at $\theta = 90°$ may be shown to be

$$S_r = C U_2 V - K_\parallel \frac{\partial U_2}{\partial r} \sin^2\psi = \frac{1}{r^2} \int_o^r K_\perp \frac{\partial^2 U_1}{\partial \theta^2} dr$$

which is an integral of (72) when two large terms on the left hand side have cancelled and a factor C omitted in the original work has been included. We see that the perpendicular streaming into the equatorial plane integrated out to the point of observation is responsible for the radial flow at that point. Note that we have suppressed radial streaming due to energy change in U by keeping C constant. Jokipii and Parker (1968) deal with the equivalent approximation for energy space gradient induced streaming.

However section 6 has adequately dealt with the physics involved.

Turning now to the experimental evidence, Sarabhai and Subramanian (1966) suggested that the marked excess in 5303A coronal line activity at northern heliolatitudes might lead to a north to south gradient in cosmic ray intensity. Following a suggestion of Marsden (1967), Hashim and Bercovitch (1972) showed that the $\underline{B} \times \nabla U$ drift due to a north to south gradient showed up in neutron monitor and meson telescope data as a sector structure correlated effect in the diurnal variation. For example, for $\underline{B}$ outwards, $\nabla_{\perp}U$ yields a flow 45°E of the earth-sun line. Hashim and Bercovitch found $G_{\theta} = 5.5\ R^{-0.6}$%/AU. Newkirk and Lockwood (1982) represent a recent work attempting a direct measurement of the latitudinal gradient by correlating cosmic ray activity with solar activity. They find a decrease of intensity with increasing latitude, measured in a heliomagnetic coordinate system that they define and for periods before and after the 1969-1970 reversal of solar field polarity.

A symmetric, rising gradient is consistent with measurements of the second harmonic of the neutron monitor and meson telescope diurnal variations since the time of maximum observed is, on the average, perpendicular to the interplanetary field direction in the ecliptic plane (Quenby and Lietti, 1968). These authors found rising gradients consistent with a modulation factor $M \sim 2,4$ GV which is about double that mentioned in section (7.2). However, Nagashima et al (1971) claim that the semi-diurnal data is best interpreted as a pitch angle distribution with maximum intensity at 90° pitch angles.

Direct measurement of a gradient rising between 0° and 16° northern solar latitude is provided by McKibben et al (1979) using Pioneer 10 and 11 data on the anomalous helium component between 11 and 20 MeV/nucleon. They found a 2-3% per degree latitude gradient at 4.75 AU. Roelof et al (1981) have analysed Voyager 1 and 2 and IMP-8 data relevant to distances between 2 and 5 AU and find transient gradients at $\gtrsim$ 30 MeV/nucleon which are directed north-south, south-north or are U-shaped and with magnitude 1 to 5% per degree. All these experimental determinations of the latitudinal gradient are consistent with values which would produce $\sim$ 100% modulation between the solar poles and the solar equatorial plane.

Inward or outward radial streaming anisotropy can also indicate the three dimensional aspect of modulation. Rao et al (1967) found a 0.18 (± 0.05)% outward streaming in the 7.5-45 MeV range which seems unlikely to fit any reasonable spherically symmetric model. For example the computations of Urch and Gleeson (1972) and of Dyer et al (1974) show inward streaming at low energies. Furthermore, integral measurement of protons at > 360 MeV by HEOS-I (Dyer et al 1974) showed a 0.3% inward streaming. These data cannot be reconciled with the spherically symmetric model which gives an outward streaming at > 1 GeV. Quenby and Hashim (1969) suggest that a small outward streaming $\sim$ 0.02 of the corotation effect is consistent with the 1964 neutron monitor data while for the long term ionisation chamber data, an outward streaming varying between 0 and 0.3 of the corotation amplitude (i.e. up to 0.12%) and depending on the phase of the solar cycle can occur. Quenby and Hashim interpret the relative variation of ξ_r and ξ_{ϕ} in terms of a variable ratio $K_{\perp}/k_{\parallel}$ with a rising, off-ecliptic gradient driving particles into the equatorial plane. Note that Forbush and Beach (1975) interpreted the same data in a different manner and the difference in phases in the asymptotic directions implied is probably due to the introduction of an empirical diurnal wave temperature correction by Quenby and Hashim.

Use of the earth's excursion of ± 7.25° in heliolatitude in a year which can give rise to an annual and semiannual wave in cosmic ray intensity enabled Antonucci et al (1978) to find a symmetric falling gradient pre-1969, a rising symmetric gradient after 1969, a southward directed perpendicular gradient in 1959-1969 and after polar field reversal in 1969-1971 a north pointing cosmic ray gradient. The symmetric gradients found by these last authors do not fit the Quenby and Hashim predictions but are explained by the theory of Jokipii and Kopriva (1979) which we will discuss later. Swinson and Kananen (1982) employ sector-structure correlated changes in the ecliptic plane component of the diurnal variation seen by cosmic ray detectors to confirm the direction switch of the antisymmetric gradient seen be Antonucci et al (1978). Swinson and Kananen emphasise the dominance of this one-way gradient over the symmetric gradient and point out that the Jokipii-Kopriva predictions are only satisfied if the earth is located predominantly above the cosmic ray equator so that it is in a region where the gradient is pointed southwards before the sun's field reversal and pointed northwards afterwards.

Space measurements have been made on the relativistic particle anisotropy at large radial distances in a plane at right angles to the earth-sun line. Between March and November 1974, Pioneer 10 at 6 → 6.8 AU (Axford et al, 1975) found an azimuthal anisotropy $\xi_\phi \approx 0.59 \pm 0.18\%$ and a north to south streaming anisotropy $\xi_\theta \approx 0.25 \pm 0.08\%$ for T > 480 MeV/nucleon. Some part of ξ_θ could not be correlated with the sector structure $\nabla U \times \underline{B}$ effect and this residual value of $\xi_\theta \approx 0.11\%$ could be explained by an asymmetric latitude dependence of the modulation. Its sign agrees with that measured by Swinson and Kananen and Antonucci et al after 1969. One problem with the data of Axford et al (1975) is that the equivalent detector onboard Pioneer 11 at 1.1 → 2.7 AU found $\xi_\theta \approx 0$ at a time in 1973 when Swinson and Kananen (1982) found definite evidence for a northward gradient.

Concerning the local control of the position of the neutral sheet in the IMF and its variability and hence the consequent variations in the plane of symmetry of the cosmic ray intensity, the analysis of Moussas and Tritakis is interesting (1982d). These authors point out that an analysis of sector structure data implies that in 1974-1977, there is an influence of the north solar pole coronal hole at all latitudes, agreeing with the southward displacement of the current sheet predicted by Rosenberg (1970) (see also section 2). It must be said, however, that rather less than 50% of the polarity data of Moussas and Tritakis are in good statistical agreement with the Rosenberg effect model for the dominant sector structure polarity Again this emphasises the importance of local disturbances and suggests that in-ecliptic measurements have little hope in establishing the overall 3-dimensional modulation configuration or solar wind pattern.

Recently streaming measurements have been made by Pioneer 10 out at 11-15 AU (McDonald and Forman 1981; Forman and McDonald 1981) in the plane perpendicular to the earth-sun line. For the 30-56 MeV proton energy range, the anisotropy is ∿ 7% which, if anything, is above the predicted corotation value and imples $\lambda_{\parallel} = 16 \pm 5$ AU and $K_\perp \leqslant 0.007\, K_{\parallel}$ when interpreted in terms of (70) and the measured gradient. These values are respectively much higher and much lower than the Moussas et al (1982b) computational values for 5 AU, but could be interpreted as revealing a much smoother IMF at these larger distances. An alternative explanation put forward by Forman and McDonald is that there is an undetected 7% inward streaming which would allow $K_{\perp} \gtrsim 0.05\ K_{\parallel}$ and be qualitatively consisten with the Dyer et al 1974 observation. However such a streaming does not fit with the approximate calculations of Quenby and Hashim (1969) who find that an increase in S_r corresponds to a decrease in S_a. Anisotropy measurement of the anomalous 10-56 MeV/nuc He component yields a streaming $\sim$ 10% of the corotation value and allows $\lambda_{\parallel} \approx 5$ AU and $K_{\perp} > 0.05\ K_{\parallel}$ for no S_r. Alternatively, if $K_{\perp} < .007\ K_{\parallel}$ there would have to be an undetected outward radial streaming of 2%. The authors believe their results to be consistent if there is significant but different radial streaming of the proton and helium component.

In contrast to these 12 AU measurements, the closer in data from Pioneer 10 and 11 of Axford et al (1975) yielded a ratio $K_{\perp}/K_{\parallel} = 0.13 \rightarrow 0.26$ based on the reduction of corotation amplitude in (70c). These results are more in accord with the quasi-linear and numerical simulation values of the diffusion coefficients.

9. The Anomalous and Low Energy Components - Experimental Evidence

Below 50 MeV/nucleon there are various puzzling features of the near-earth spectrum that may or may not require 3-D modulation models for their explanation. In particular, the cosmic ray spectrum at these energies is characterised by anomalously high fluxes of helium, nitrogen, oxygen, neon and possibly iron (Garcia Munoz et al, 1973; Hovestadt et al 1973; McDonald et al, 1974; Klecker et al 1977) which have been observed since 1972. There is also a steep turn-up in the spectrum at the lowest cosmic ray energies (Figure 11 and Mason et al, 1977). The anomalous composition would correspond to an overabundance of 5-20 times normal cosmic ray or solar system abundances, provided carbon at the same energy is taken to be entirely of galactic origin.

It has been pointed out by Fisk et al (1974) that the enhanced elements have first ionisation potentials higher than hydrogen and that they may exist in interstellar space as neutral atoms. However interpretation of 1977-78 Voyager observations by Webber et al (1979) suggest that some anomalous intensity increase is found for C, Mg and Si, although at definitely lower levels than for the previously mentioned elements. These additional enhanced elements have lower first ionisation potentials. Anomalous He and O exhibit a 27-day resonance tendency, seen in association with $\gtrsim$ 0.5 MeV proton enhancements and also a Forbush decrease has been observed in anomalous O (Webber et al 1979; Garcia Munoz et al 1977b). A positive radial gradient has been observed in He and O in the 10-20 MeV/nucleon energy range of magnitude $\sim$ 15%/AU (Webber 1979).

Indirect evidence for a singly charged state for anomalous helium has been provided by McKibben (1977). He considered the phase lag of low energy cosmic rays with respect to particles of a higher energy, assuming it to be only a function of velocity and rigidity. There is some evidence from 1974-75 IMP-8 data for 11-20 MeV/nucleon helium intensity changes being in advance of those of 51-95 MeV protons which would be at an equivalent magnetic rigidity if the He was fully ionised. Instead, the He behaved more like singly ionised particles, corresponding to a higher rigidity. McDonald et al (1979) studied $\lambda_{\parallel}$ as deduced from gradient and spectral measurements, based upon the approximation CUV $\simeq K_{rr}\,\partial U/\partial r$. They found that their data was better ordered if He $\lesssim$ 50 MeV/nucleon is singly ionised. Incidentally, the values of λ_r obtained at $\sim$ 200 MV, i.e. $\lambda_r \sim .04$ AU, are in reasonable agreement with the numerical simulation discussed in section 4. Anisotropies are unlikely to be so large that the approximation exmployed is seriously wrong.

Paizis and von Rosenvinge (1981) extended the work of McKibben (1977) with a plot of the time lag with respect to higher energy particles of the various charge species for protons, ordinary and anomlous helium against βR. They found that 8-22 MeV/nucleon He did not fit the curve, whatever the assumed charge state. In fact the time lag for these particles was very short and the authors concluded that the model of O'Gallagher (1975) which takes into account hysteresis in symmetric modulation theory and upon which the $R\beta$ plot is based does not order the data sufficiently for deductions to be made concerning the He charge state.

Garcia Munoz et al (1981) re-examined old data to show that the anomalous He component was not present before the 1969-71 solar field phase reversal in strength comparable to that seen in 1977. Hence drift motion involving a 3-dimensional model is favoured for the origin of the anomalous component.

Webber et al (1981) made a comparative study of the radial gradients of anomalous helium and oxygen to obtain similar gradients $\sim$ 15%/AU out to 15 AU in a time of relatively little temporal change in the cosmic ray intensity. Becuase of the differences in Compton-Getting factor, larger for O than He, this similarity turns out to be more consistent with the single ionisation of these two charge components. Another point noticed by the authors is that during temporal changes by a factor 10 in the modulation level, the radial gradient remained the same. The authors discuss the possibility that K_r changes by this same factor 10, although such evidence as we have on variation in the power spectra of magnetic fluctuations does not support such a change. Alternatively, Webber et al (1981) suggest on the basis of a simple, spherically symmetric model with K_r = constant, that alterations of the effective boundary depth by several hundred AU could account for the observation. Again an appeal to off-ecliptic effects may be helpful.

von Rosenvinge and Paizis (1981) discuss the large amplitude of the modulation exhibited by anomalous He relative to protons at a similar rigidity. They demonstrate that it is the differences between the spectral slopes of these components which may account for the observation. Thus the protons show a positive spectral shape at the relevant rigidity which allows decelerated particles to partly compensate for modulated, lower energy particles. The He atoms have a flat spectrum, so less compensation takes place.

The turn-up in the low energy spectrum seen below $\sim$ 5 MeV/nucl. may be related to corotating particle streams first seen in 1965 (Fan et al 1965; Bryant et al, 1965), rather than galactic cosmic rays. Proton streams exhibiting a 27-day recurrence tendency are found to correlate in time with the passage of high-speed solar wind streams. Near-sun and deep space probe observations have established the likelihood of an interplanetary acceleration mechanism for these particles. Gradients have been seen; ∿ 350%/AU between 0.3 and 1 AU, ∿ 100%/AU between 1 and 3 to 5 AU and negative gradients out from 4 to 9 AU (Van Hollebeke et al, 1978). Some compositional changes have been noticed within the tendency for these events to be enhanced in the region of the forward and backward shocks associated with fast streams. The H/He ratio has been seen to increase by ∿ 100 at the forward shock (Barnes and Simpson, 1976). Nevertheless, Gloeckler (1979) finds relatively little variation in the composition during these events as seen in near-earth data. The most striking difference from prompt solar flare event particle abundances lies in the carbon:oxygen ratio. It is consistently less than 0.8 for flare events but lies between 0.9 and 1.5 for corotating particle streams. Hence flares are unlikely to be the source of the corotating particles. However the composition of these particles resembles that of the solar corona and presumably that of the solar wind, especially the He, C, Fe and O components (Gloeckler 1979). Study of the spectra of these species, together with protons, in recurrent events show the distribution function to be well fitted by an exponential in velocity. Moussas et al (1982a) review current evidence on these streams to demonstrate a statistical, rather than shock-associated acceleration to be most likely.

10. The Low Energy Components in the Context of Spherically Symmetric Modulation Theory

Although the review of evidence in section 9 concerning the anomalous and low energy spectral components suggested some appeal to 3-dimensional modulation models, it isnecessary to see which facts can be understood in the context of the spherically symmetric models. Such symmetric solutions of the Fokker-Planck do not necessarily comprise a unique set of explanations and some off-ecliptic effects may need to be added to the models.

Concerning the anomalous component, Fisk (1976c) has discussed the requirements upon modulation theory including only the symmetric terms in the Fokker-Planck equation without acceleration which would allow into the heliosphere sufficient galactic particles to provide the high He, O and N fluxes at 10 MeV/nucleon. By assuming $\underline{\underline{K}} \propto vg(R,r)$ i.e. a function of velocity and magnetic rigidity, and by making a reasonable estimate of the amount of electron modulation at the same rigidity, Fisk shows the necessary interstellar fluxes to be impossibly high. He then puts forward a possible modification to the diffusion process whereby K_{rr} is controlled by the lifetime of magnetic traps in which particle mirroring takes place and suggests this circumstance can occur beyond 1 AU. Such particle trapping has not been obvious in the computations of Moussas et al (1982b), performed at 5 AU.

Alternatively, Fisk et al (1974) suggest that the anomalous component originates in the part of the neutral interstellar particle population which has a high first ionisation potential. These particles can penetrate into the heliosphere before suffering ionisation by charge exchange with the solar wind or because of the enhanced UV radiation levels. After becoming singly ionised, they would be accelerated by the statistical mechanism of section (4.4) and also suffer outward convection in the solar wind. As the particles gain in energy, some diffuse back because of their relatively high magnetic rigidity. Klecker (1977) fits modulation data taken both at high energies and for the anomalous component to Fisk's theory. He employs as parameters in the model, $K_r \sim 10^{22} \exp[(r-1)/30]\beta R$ cm^2 s^{-1} and $D_{pp}/p^2 \sim 3.10^{-8}$ s^{-1} at 10 MeV/nucleon with a solar cavity radius of 50 AU. The spatial diffusion coefficient is higher than the numerical estimate of section (4.1) while the energy diffusion coefficient is lower than that of (4.4). Also the fraction of singly ionised atoms from the injection process which take part in the acceleration, $\varepsilon = 2 \times 10^{-3}$ is a parameter which is simply fitted to the observations. Figure (12) shows the results of Klecker's model calculations. Thus we may conclude that the Fisk et al (1974) hypothesis has some success in explaining the anomalous component in the context of spherically symmetric modulation theory when acceleration is added. Indeed it is very difficult to believe that the process is not taking place in the IMF, to some extent at least. However because all the parameters employed in the model do not necessarily agree quantitatively with other estimates, some other elements such as off-ecliptic effects may be necessary to completely specify the physics involved.

At even lower energies, the statistical acceleration mechanism emphasised by Fisk (1976a,b) is likely to explain the quiet-time turn up of the spectra and the observed distribution functions for H, He. C, O and Fe in corotating streams. In particular, the study by Gloeckler et al (1979) based on IMP D data in the 0.15 MeV/nucleon to 8 MeV/nucleon range showed $f \propto \exp(-v/v_o)$ for these species. These authors showed that a steady state solution of the symmetric Fokker-Planck plus acceleration is of this form provided $p^2/D_{pp} \sim v$. The numerical results of Moussas et al (1982a) with $D_{TT} \propto T^{1.5}$ give just this proportionality. Gloeckler et al (1979) also point out that since the abundances are comparable to that of the solar corona, acceleration from solar wind energies without an injection threshold, as occurs in shock acceleration, is favoured.

We conclude that the evidence from section (9) is for interplanetary acceleration but is not necessarily as compelling for 3-dimensional effects as other evidence, discussed previously.

11. Three-Dimensional Modulation Models

It is clear that a comprehensive model for cosmic ray modulation must take full account of the latitude dependence of the spiral field geometry and sign of the IMF, together with the latitude dependence of the solar wind velocity and magnetic turbulence. Sufficient experimental evidence has been put forward to suggest the real existence of perpendicular to the equatorial plane gradients and flow patterns and both experimental and numerical modelling evidence demonstrate that drift motion perpendicular to $\langle\hat{\underline{B}}\rangle$ can have noticeable effects upon the near-earth cosmic ray intensity.

One of the initial problems investigated by numerical solution of the 3-D Fokker-Planck equation was that of reconciling the small, measured radial gradients with the small values of $\lambda_{\parallel}$ that both analytical theory and numerical modelling require. Fisk (1976d) employed the Jokipii (1971) estimates for the spatial diffusion coefficients to give :

$$K_{\parallel} = \frac{v r_g^2 B_o^2}{P_{xx}(k=0)} \left(\frac{\lambda_{cor}}{r_g}\right)^{3/2}$$

assuming $P(k) \propto k^{-3/2}$, λ_{cor} = correlation length = 1.5×10^{11} cm and with the factor $(\lambda_{cor}/r_g)^{3/2}$ put equal to unity if $r_g > \lambda_{cor}$. Also

$$K_{\perp} = \frac{vP_{xx}(k=0)}{2B_o^2} \left(\frac{r_g}{\lambda}\right)^{3/2}$$

again with the factor $(r_g/\lambda)^{3/2} = 1$ if $r_g > \lambda_{cor}$. Drift motion due to K_T was neglected in the Fokker-Planck and an alternating gradient modification of the Crank-Nicholson numerical technique employed. $K_{\perp}$ was kept constant with respect to r, but $K_{\perp}/K_{\parallel} \propto r$ at < 2 GV due to the r_g dependence. At > 2 GV, particles scattered into the equatorial plane from the regions of easy access along near-straight polar field lines preferentially at small r. Figure 13 shows the results obtained and depicts the near-earth proton intensity, the radial gradient and radial anisotropy. Note the few percent per AU radial gradient, in good accord with experiment, but also the outward streaming which is a natural consequence of the easier, off-ecliptic access. This outward anisotropy is not in accord with the Dyer et al (1978) spacecraft data, but qualitatively agrees with the Quenby and Hashim (1969) interpretation of ground level diurnal variation data. Figure (14) shows the latitude dependence obtained by Fisk at 1 AU for 25 MeV and 1 GeV particles with either finite or zero $K_{\perp}$ (= K_{θ}). Note the reduction in polar gradient that can be brought about by the introduction of perpendicular diffusion and the consequent enhancement of the near-earth intensity. Alaniya and Dorman (1977) performed a similar calculation to that of Fisk (1976d) and demonstrated that acceptably small radial gradients can be obtained provided the diffusion is anisotropic ($K_{\perp}/K_{\parallel} < 1$) out to a distance ∿ 8 to 16 AU.

Dorman and Milovidova (1973) solved the Fokker Planck (72) allowing $K_{\parallel} \propto K_{\perp} \propto \eta^{-0.6}\, r^{1/2}$ where η is the monthly sunspot number evaluated as a function of solar latitude. For the years 1958 and 1964 peak intensity was found at the solar poles, but a north to south gradient occurred across the equatorial plane.

Cecchini and Quenby (1975) were concerned to explain the inward streaming at 1 GV (Dyer et al 1974) in terms of a $K_{\parallel}$ and $K_{\perp}$ latitudinal variation such that diffusion was at a minimum opposite the zones of maximum sunspot activity. Inward streaming in the equatorial plane and at high solar latitudes is balanced in their model by outflow opposite the sunspot zones. These authors integrated the Fokker Planck (72), which still neglects K_T and allowed $K_{\perp} \propto K_{\parallel} \propto rg(\theta)$ for r > 1 AU where $g(\theta) = 1 + \cos\theta\,(5\cos^2\theta - 3)$ and $K_{\parallel}$ at 1 GV and 1 AU took the value 2.10^{22} $cm^2\ s^{-1}$. The model successfully explained the streaming observation of Rao et al (1967) and Dyer et al (1974), but despite the over-large $K_{\parallel}$ value adopted, still gave rather large radial gradient, e.g. 20%/AU at 100 MeV. This last fact is likely to be physically due to the necessity in the model for bringing particles in through the equatorial plane, unlike in the above Fisk model where input was predominantly at high latitudes.

A fairly recent and clearly important advance in modulation theory has been the realisation that drift motion in a mainly unidirectional $\hat{\underline{B}}$ is important. The basic formula for the inclusion of this K_T effect in the Fokker Planck have been given in sections (3) and (4.3). That is we add to equation (72) a term given by (42). At the Kyoto Cosmic Ray Conference, three groups provided numerical solutions of this full Fokker-Planck (still without acceleration however), including the K_T term (Moraal et al, 1979; Jokipii and Kopriva, 1979; Alanyia and

Dorman, 1979). The basic difference between the models of Moraal et al and Jokipii and Kopriva lies in the importance of drift in the neutral sheet.

Moraal et al in fact put $K_T = 0$ and $\langle \underline{v}_D \rangle = 0$ at the neutral sheet on the grounds that we do not know if special trajectories are possible as in the earth's magnetic tail. We have already noted the limited information on the actual configuration of the neutral sheet contained in the analysis of Villante et al (1979). The numerical solutions of Moraal et al always yield a rising perpendicular gradient away from the equator, as illustrated by Figure 15. This figure shows pre- and post-1969/70 reversal conditions at two energies, based upon the diffusion coefficients

$$K_{\parallel} = K_{\parallel}^{\circ} \beta R (1 + (\frac{r}{r_e})^2) \qquad R > 0.4 \text{ GV}$$

$$= K_{\parallel}^{\circ} \beta (1 + (\frac{r}{r_e})^2) \qquad R < 0.4 \text{ GV}$$

$$K_{\perp} = K_{\perp}^{\circ} \beta (\frac{r}{r_e})^2 (1 + \sec \psi)$$

with $K_{\perp}^{\circ} = 1.2 \times 10^{20}$ cm^2 s^{-1}, $K_{\parallel}^{\circ} = 1.2 \times 10^{22}$ cm^2 s^{-1} for weak modulation conditions and $K_{\parallel}^{\circ} = 3.0\ 10^{21}$ cm^2 s^{-1} for strong modulation conditions. Apart from the positive latitudinal gradient, we notice the opposite drift effect on protons and electrons, because of the sign dependence of K_T. Pre-1970, the protons are depleted relative to the electrons because the former come in via the equatorial plane and drift poleward while the later come in via the poles. Post-1970 the situation is reversed. This puts on a quantitative basis the explanation of the difference in the electron and proton hysteresis loops noticed by Korff and Mendell (1977) and Rockstroh (1977) and discussed by Jokipii, Mendell and Quenby during the course of the Plovdiv Cosmic Ray Conference (Quenby 1977). The very large modulation seen for the situation of North Field IN, case B, is because particles are swept out both by the solar wind velocity and the $\langle v_{\parallel} \rangle$ drift since $\langle v_{\parallel} \rangle$ drift is in the (curl B)$_{\parallel}$ direction and the only mode of entry is by diffusion in the equatorial plane. Figure 16 for cosmic ray protons illustrates a numerical study of the fraction of near Earth particles that originate at different latitudes on the solar cavity boundary. We notice that for north field IN where the drift is out of the equatorial plane there is a tight grouping about $\theta = 90°$. This contrasts with the wide spread of entry points for north field OUT.

Jokipii and Kopriva (1979) include an additional term in the K_T part of the diffusion tensor corresponding to rapid particle drift motion along the theoretical interplanetary neutral sheet. This is

$$\langle v_D^{\text{neutral}} \rangle = (1 + \Gamma^2) \Gamma\ \delta(\theta - \frac{\pi}{2})\ (\underline{\hat{e}}_r + \frac{\hat{e}_\phi}{\Gamma}) \qquad (73)$$

where $\Gamma = r\Omega \sin \theta / V$. Motion along the neutral sheet is illustrated in Figure 17 where the migration is in the $\nabla \times \underline{B}$ direction for particles whose gyroradii include this plane. Otherwise the parameters employed are not dissimilar to those of Moraal et al (1979). In particular,

$K_\parallel = 5.10^{21} R^{1/2} \beta \text{ cm}^2 \text{ s}^{-1}$, $K_\perp = 0.1 K_\parallel$ and integrations were carried out to a similar boundary position at 10 AU. Figures 18, 19 reproduce their numerical results for q A positive, that is post-1969 and q A negative (pre-1969) where A specifies the field strength. In the figures U/U_B refers to intensity relative to that at the cavity boundary. An important point to notice is that for q A positive, the radial gradient in the equatorial plane, $\theta = 90°$, is very low for 70 MeV protons. The authors demonstrate that this result does not depend very much on the diffusion coefficient, $K_\parallel$ adopted, which can be very small. Hence there is no problem with reconciling more recent radial gradient measurements and numerical values of $K_\parallel$ with the model.

Concerning the pre-1969/70 results for q A negative, the falling gradient away from the equatorial plane fits with several measurements discussed in section (8). Also the Dyer et al (1974) inward streaming which was at its largest early in the year fits qualitatively with the flow pattern expected from a falling latitudinal gradient. Such an inflow pre-1969 also satisfies the work of Levy (1975), based on neutral sheet motion which effectively fits into the Jokipii and Kopriva formulation because of the latter's use of (73). Levy pointed out that the Forbush and Beach (1975) analysis of the 22-year wave was consistent with $\nabla \times \underline{B}$ directed outflow post 1969/70 and inflow pre 1969/70. Also an approximate solution of the 3-dimensional modulation equation yielded the result that meson detectors would see this effect enhanced over that seen at lower energies by neutron monitors, again in qualitative agreement with experiment. The effect of the scale size of neutral sheet distortion on the idealised trajectories has not yet been discussed however.

One point concerning the Jokipii and Kopriva prediction of Figure 19 for q A negative which might cause concern lies in the large magnitude of the radial gradient at $\theta = 90°$. It is worth remembering, however, that early measurements of the gradient by direct means and indirect means (O'Gallagher 1967; O'Gallagher and Simpson, 1967, for Mariner 4; Lezniak and Webber 1973 for Pioneers 8 and 9 and Bercovitch (1971) using neutron monitors) all give gradients in the $\gtrsim$ 10%/AU range for pre-1970 data at relatively high energy. However Anderson (1968) provides a counter example of a gradient measurement in 1965 yielding < 10%/AU.

The steep rise near the boundary in all the plots of Figures 18 and 19 seem unphysical and could result from an inability to model this region well. Jokipii and Kopriva (1979) compute the radial anisotropy arising in the q A positive situation and find $\sim 0.2 \rightarrow 0.3\%$ at $1 \rightarrow 2$ AU. The authors emphasise the existence of a broad, interior plateau in the low energy, equatorial region which is connected to the inner and outer boundaries by thin boundary layers of rapidly changing intensity as mentioned above. It seems possible that the outer layer at least is an artefact of the model representing a sudden switch from entry along the near equatorial plane or near polar field lines to transverse diffusion under $K_\perp \nabla_\perp U$ as the boundary is approached and the easy motion path parallel to $\hat{\underline{B}}$ get too long. Actually the one situation where this qualitative explanation will not work is for q A negative, $\theta =$ 90° where inward motion along the neutral sheet is important. However for this case, the outer boundary layer disappears (Fig.19). In the model, $K_\parallel$ was maintained constant with distance, which is clearly not the case, while r_∞ was artificially close to the sun. A more distant boundary and a more realistic variation of $K_\parallel$ with r could probably reduce the boundary layer gradients to reasonable values.

Jokipii and Thomas (1981) extended the above model of Jokipii and Kopriva to include an approximate treatment of the wavy current sheet effect on the flow and intensity (equation 6, section 2). They are concerned to incorporate a possible solar cycle dependent change in the current sheet tilt in the model and thereby produce an explanation for the 11-year cycle independent of the constancy of the magnetic fluctuations and solar wind speed parameters which usually specify the modulation level. They are able to use a reasonable variation in the tilt angle, between 10° and 30°, to give the observed solar minimum to maximum cosmic ray variation. Higher tilt means more path length in the neutral sheet and greater modulation. The observed much flatter cycle variation for q A positive as compared with q A negative is reproduced in the theory. Jokipii and Thomas obtain their results by neglecting $K_{\parallel}$, based on the previous, Jokipii and Kopriva (1979) work which showed that approximately similar results could be obtained with or without the inclusion of parallel diffusion. It is important to realise that the nuetral sheet motion is crucial to the exact working of these 3-D models and if subsequent investigation of particle trajectories in a realistic, jagged neutral sheet produce a much lower $\langle v_D{}^{neutral} \rangle$ than used in (73), difficulties will appear with the models, especially for q A negative.

In conclusion, it is worth saying that the diffusion plus drift model for modulation, including neutral sheet motion, promises to satisfy a variety of evidence, although some conflict in the experimental data does not permit a certain conclusion on the model's validity as yet. In its favour, the Jokipii-Kopriva-Thomas model :

(a) is required because numerical computation supports the inclusion of guiding centre drift;

(b) is required because of the disappearance of the high latitude sector structure;

(c) fits with the required low $K_{\parallel}$ values;

(d) fits the observed electron-proton hysteresis;

(e) fits some inward streaming observations;

(f) fits some perpendicular gradient observations;

(g) may explain the 11-year cycle;

(h) may throw new light on early and possible large radial gradient measurements.

We have mentioned some perpendicular gradient observations (e.g. Newkirk and Lockwood, 1982) and streaming observations (e.g. Quenby and Hashim, 1969) which do not seem to fit the drift model. Also the local solar activity control of IMF conditions at 1 AU has been emphasised, making many experimental checks difficult. The ISPM mission will surely provide crucial evidence on the role of drifts by getting away from these local factors. However it would seem that ISPM will be launched in a q A negative epoch and have little hope of seeing near galactic cosmic ray conditions over the solar poles.

12 Effects at the Boundary of Modulation

For completeness we briefly discuss two other aspects of modulation. Regarding the first, which is the helio-cavity boundary effect, the reader is recommended to study the excellent review of Axford (1971). The heliosphere is defined by the confinement due to the interstellar medium. Fahr et al (1981) may be consulted as a recent investigation of the combined effects on stopping the solar wind exerted by the interstellar ram pressure. This pressure is made up of the interstellar magnetic field pressure, the momentum of the interstellar plasma in motion relative to the solar system, the interstellar plasma thermal pressure and the momentum exchange due to the ionisation of interstellar atoms as they penetrate near to the sun. A weak shock is expected at $\sim$ 100 AU, constituting a heliocavity boundary not inconsistent with positions suggested by cosmic ray gradient measurements. Babyan and Dorman (1979) present one of a number of calculations in the literature, based upon spherically symmetric modulation, in which the additional slowing down effect of cosmic rays on the solar wind is included. For likely, low values of the interstellar neutral hydrogen density, i.e. $N_H \sim 0.1\ cm^{-3} \rightarrow 0.5\ cm^{-3}$, the cosmic ray effect exceeds in magnitude that due to charge exchange with the neutral gas and the solar wind is slowed by a factor 2 in velocity in the 20-100 AU region, the exact scale size of the deceleration being dependent on the form and energy density of the galactic cosmic ray spectrum.

The problem of the access of interstellar cosmic rays to the IMF lines has been discussed by Schatten and Wilcox (1964) and Nagashima and Morishita (1979). These authors believe that access is easier if the solar wind and interstellar magnetic fields are nearly parallel, rather than anti-parallel, and that 22-year cycle effects in the diurnal, long term and anomalous component variations can be explained by the efficiency of this transfer process. Morfill and Quenby (1971) have computed in detail the ability of energetic charged particles to transfer across a tangential discontinuity representing the field boundary between the geomagnetic tail and the solar wind flow in the magnetosheath. The implication of their results is that the relative orientation of the two sets of magnetic field lines has little effect on the efficiency of particle transfer, provided the dimensions of the interface are on a sufficiently large scale. Significant turbulence at the heliosphere boundary would invalidate all these models.

Speculation about acceleration of cosmic rays at the cavity boundary is a topic which has been revived recently by Eichler et al (1981). Attention was drawn to shock acceleration at the supersonic to sub-sonic flow transition at this boundary by Jokipii (1968). Eichler et al however point out that a favourable configuration for acceleration with the shock normal nearly parallel to $\underline{B}$ probably occurs only within a few degrees of the solar poles because the angle between these directions is the same as ψ, the 'garden hose' angle we have previously used. The acceleration is favourable because the injection speed is low. This occurs because the speed needed by a particle to escape upstream from a shock after reflection, or to overtake a shock from a downstream position, goes down as ψ decreases. Eichler et al suppose ionised interstellar gas to constitute the injection material at the solar poles for the anomalous component. This component then drifts down to the solar equator provided the field is orientated in the post 1969/70 manner. Before this phase reversal, drift is polewards and hence the observed presence of the anomalous component in the 1970s only and not pre-1969 is explained. An energy of roughly 240 MeV/charge is required to enable the particles to drift the full latitude range against the $-\underline{V} \times \underline{B}$ electric field. Hence this energy constitutes a minimum requirement on the boundary acceleration mechanism. We see that Eichler et al (1981) have provided a variant of the Fisk et al (1974) mechanism for the anomalous component which incorporates the 3-dimensional aspects of modulation. However this boundary acceleration may not be a unique explanation of the 22-year wave aspect of anomalous spectral observations. Poleward drift of the newly created ions, either during statistical acceleration or during subsequent modulation, in the Fisk et al process under pre-1969 conditions may be an equally good reason for the lack of an observed anomalous flux at that epoch.

Jokipii and Levy (1979) have questioned from another viewpoint the spherically symmetric boundary conditions commonly taken at the heliosphere boundary for the galactic spectrum. They note that the $-\underline{V} \times \underline{B}$ electric field produces an equator to pole potential difference of 218 MeV using equation (6). This could have a marked effect on cosmic rays entering with a comparable energy, as is evident from our discussion in section (5.2) on the Kota process. Jokipii and Levy appear to neglect any additional boundary potential Φ_B as discussed by Erdős and Kota (1978) which could be due to a curl $\underline{E}$ connected with a reconnection process at the boundary. They instead suppose a vacuum outside so that cosmic rays from infinity are modulated at the boundary by an amount given by Liouville's theorem for propagation in the electrostatic potential defined by $-\underline{V} \times \underline{B}$ with $\underline{B}$ given by (6). Alternatively, Jokipii and Levy consider a situation where some IMF lines connect to the interstellar medium via exit points where the solar wind flows out beyond the shock transition downwind with respect to the interstellar medium wind flow. It is expected that $\underline{E}.\underline{B} = 0$ along each field line. Hence some parts of the heliosphere are at the same electrostatic potential as the interstellar cosmic ray component at great distances, while other parts are not directly connected to interstellar space and may be at the potential defined by $-\underline{V} \times \underline{B}$. On either model, intensity variations at the boundary comparable to the total depth of modulation may occur.

13. Dynamic Modulation

We have neglected to discuss explicitly the cause of the Forbush decrease in this review, partly because it is likely to be the result of transient changes in the solar wind connected with passage of special discontinuity configurations and partly to keep the amount of material under reasonable control. Lockwood (1971) provided a review of Forbush Decrease observation and theory which can be consulted. It is necessary, however, to mention the attention which has again been drawn to the hypothesis that the long-term modulation is actually made up of a series of Forbush Decreases. McDonald, Trainor and Webber (1981), observe from their Pioneer 10 and Voyager data that between June and September 1980, the level of the long term variation showed a decrease of 18% in the > 200 MeV proton flux at both 9 AU and 23 AU and that this reduction was achieved in a series of four Forbush Decreases. Each of these decreases had a sharp onset, thus requiring a barrier mechanism such as Parker's (1961) shock front or Quenby's (1971) tangential discontinuity as part of the explanation. However the minimum intensity in the event was seen when the solar wind had moved on 2-3 AU. It is suggested by McDonald et al that some additional cause is required to explain this minimum and that Forbush decreases are an important component in the long term modulation.

Preliminary theoretical investigations by Fisk Lee and Peko (1981) and Alaniya, Guschina and Dorman (1981) deal with time dependent solutions of the Fokker-Planck transport equation with increased particle scattering downstream of regions behind flare produced shock fronts. The former authors neglect drift and take each decrease as being due to a factor 10 decrease in K_r for 8 days. The latter authors allow K_r to be a function of r, t, θ depending upon the mean monthly sunspot area and green coronal line intensity at the appropriate latitude and also include drift motion. Fisk et al (1981) produce a predicted time dependence of the cosmic ray intensity which is more jagged nearer sunspot minimum and less jagged at solar maximum when several shocks are present in the heliosphere at the same time. Hysteresis effects occur between the high and low energy proton intensities. Alaniya et al (1981) claim, on the basis of a few preliminary computations, that a drift particle flux is a necessary feature of the model in order to explain the solar cycle.

14. Conclusions

As a brief conclusion to this review, we first re-iterate the point that the basic process of modulation has been understood for some time. What is needed for detailed understanding is a better appreciation of the magnitude of the various effects summing to give the total mechanism.

Advances in the determination of the magnitude of the transport coefficients and their heliospherical distribution have been made, but further progress depends to a great extent on information potentially available from an out-of-ecliptic survey and analysis of data at $\gtrsim$ 20 AU. The data required will come mainly from the magnetometer and plasma probes on the spacecraft going to these places.

The necessity to include drift motion, statistical acceleration and time dependent effects in the solution of the transport equation seems established. Computational problems involved become more formidable but must be faced if a full understanding of modulation is to be achieved.

Specification of the boundary conditions in terms of particle fluxes remain a difficulty which the experimentalist must help to solve. In particular the source of the anomalous component and low energy turn up should be known in terms of position and distribution function in the inner solar system as an input to the Fokker-Planck equation solution. Also evidence on the cavity boundary flux, possible position dependent modulation and acceleration at this interface is required, together with more theoretical study of the physical mechanisms.

In terms of the impact of modulation studies on astrophysics in general or other branches of Solar-Terrestrial Physics, the detailed investigation of the plasma physics of particle transport is important. Studies carried out in the IMF on the ability of theory to describe particle transport in position space and energy space are probably unrivalled in their detail in the field of natural plasmas. Application of the lessons we are learning in the IMF to magnetospheric and astrophysical problems should be most profitable.

Unfortunately we are little nearer the complete demodulation solution. That is to say we cannot tell the astrophysicist yet exactly what the interstellar cosmic ray flux is at all energies and for all charge species. More penetrating deep space probes may be the only answer!

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FIGURE CAPTIONS

Figure 1 A representation of the Heliosphere current sheet as seen by an observer 30^{o} above the equatorial plane and 50 AU from the sun for $V = 4 \times 10^{7}$ cm s^{-1} and a tilt angle $\alpha = 15^{o}$. (After Jokipii and Thomas, 1981). The figure is 25 AU across.

Figure 2 Solar wind speeds averaged into 15^{o} latitude intervals for six month periods are plotted against time in (b) and (c). Single rotation estimates of polar coronal hole areas derived from photospheric observations are plotted for the north polar hole in (a) and for the south in (d). The slowing of the wind above a $\pm 30^{o}$ latitude in 1978 is matched by a narrowing of the polar holes. (after Coles et al, 1980).

Figure 3 The shape of the current sheet as extrapolated from the location of the boundary crossings, the orientation of the local normals of the current sheet and the longitudinal extension of the unipolar regions of the interplanetary magnetic field. Solid lines correspond to the region of direct knowlege of the current sheet by HELIOS observations. (after Villante et al. 1979).

Figure 4 Transverse power spectral density in interplanetary magnetic field turbulence at different mean radial distances from the sun (R in AU). (After Thomas and Smith 1980 b).

Figure 5 Power spectral density of fluctuations in $|\underline{B}|$ of the IMF at various values of R (AU). (After Thomas and Smith 1980 b).

Figure 6 A simple approximation to the pitch angle diffusion coefficient D (η) suggested by Völk (1973). The dotted curve indicates the quasilinear result D_{ql}. The actual D (η) is taken equal to D_{ql} (η) for $|\eta|>|\eta_o|$ and equal to D (o) for $|\eta|<|\eta_o|$. η is cosine pitch angle in the notation of Völk.

Figure 7 The kinetic energy spectrum of the differential intensity j_T (r,T) for a monoenergetic galactic spectrum of protons at infinity ($U_p \rightarrow N_g$ $\delta(p - p_o)$ as $r \rightarrow \infty$). The kinetic energy of injection T_o is equal to the rest energy E_o. The diffusion coefficient $K = K_c$ $pr^{1.5}$ and Vr/K (r,p_o) = 0,01, 0.1 and 1.0. (after Gleeson and Webb. 1979).

Figure 8 Flow lines in the (r,p) plane for a monoenergetic alactic spectrum of particles at infinity ($Up \rightarrow N_g$ $(p - p_o)$ as $r \rightarrow \infty$). The diffusion coefficient $K = K_c$ $pr^{1.5}$ and Vr_e $/K(r_e,p_o)$ = 0.1
The flow lines are shown by the full lines whereas the locii $<\dot{r}> = 0$, $<\dot{p}> = 0$ and the critical curve are shown by broken lines. (after Gleeson and Webb).

Figure 9 To demonstrate the insensitivity of the near-Earth proton spectrum to the form of the low energy galactic spectrum with $K = 6 \times 10^{21} \sqrt{r}\, P \beta\, cm^2\ s^{-1}$. The force-field parameter $\phi(r = 1AU) = 0.14GV$ and the solar cavity boundary r_b = 10 AU (after Gleeson and Webb).

Figure 10 (a),(b) A series of essentially monoenergetic nuclei spectra in interstellar space (solid) and their resultant modulated spectra at the Earth (dashed). Figure 10a is for protons, figure 10b is for helium nuclei (after Goldstein et at 1970).

Figure 11 Differential energy spectra of hydrogen, helium, carbon and oxygen observed in the interplanetary medium near 1AU during the solar minimum in 1976-77 at times when solar flare particles were not present. Particles in the rising portions of the energy spectra are believed to be predominately interplanetary in origin. The galactic cosmic rays are observed for H above 10 MeV, for He above 60 - 80 MeV/nuc and for C and O above 30 MeV/nuc. In the intermediate energy range ($\sim$ 1 to $\sim$30 MeV/nuc) appear the anomalous 'cosmic-ray' component where helium and oxygen are highly overabundant and have 'humped' spectral shapes. At still lower energies, a quiet time turn-up is seen. (after Mason et al, 1977).

Figure 12 Comparison of model calculations with experimental results for the quiet time spectra. 1973 quiet time He, N and O data ⊕; 1973-1975 low energy N, O, Ne data ●; high energy He and O data (+). The high energy N and Ne spectra are normalized to the oxygen spectum using galactic cosmic ray abundances. Curves (1) and (2) are the calculated spectra for the galactic component and for the anomalous component, respectively. After Klecker (1977) where references to the original sources of the data are also given.

Figure 13 The near-Earth intensity spectrum (J_T), gradient (Gr) and radial anisotropy (ξ_r) for the latitude-dependent model of Fisk (1976d).

Figure 14 A plot of intensity versus polar angle θ at r = 1AU for T = 25 MeV and T = 1 GeV protons from Fisk (1976d). Note the difference between no polar diffusion $K_\theta = 0$ (dashed curves) and polar diffusion included ($K_\theta \neq 0$, full curves).

Figure 15 Solutions of modulation equation for near-Earth intensities of protons (a) and electrons (c). Polar intensity distributions for protons at 1AU are shown in (b). (after Moraal et al 1979).

Figure 16 Illustration of the fraction of near-Earth particles that crossed the boundary of the modulation cavity at a particular heliolatitude (after Moraal et al 1979).

Figure 17 Energetic, positively charged particles whose guiding centres fall within a gyration radius of the magnetic reversing layer undergoing rapid migration in the direction $\nabla \times \underline{B}$ (after Levy 1975).

Figure 18 Computed intensity (or density) of 70 MeV protons as a function of heliocentric radius r, at 10^o intervals. q A is positive (after Jokipii and Kopriva, 1979).

Figure 19 Computed intensity (or density) of 70 MeV protons as a function of heliocentric radius r, at 10^o intervals. q A is negative (after Jokipii and Kopriva, 1979).

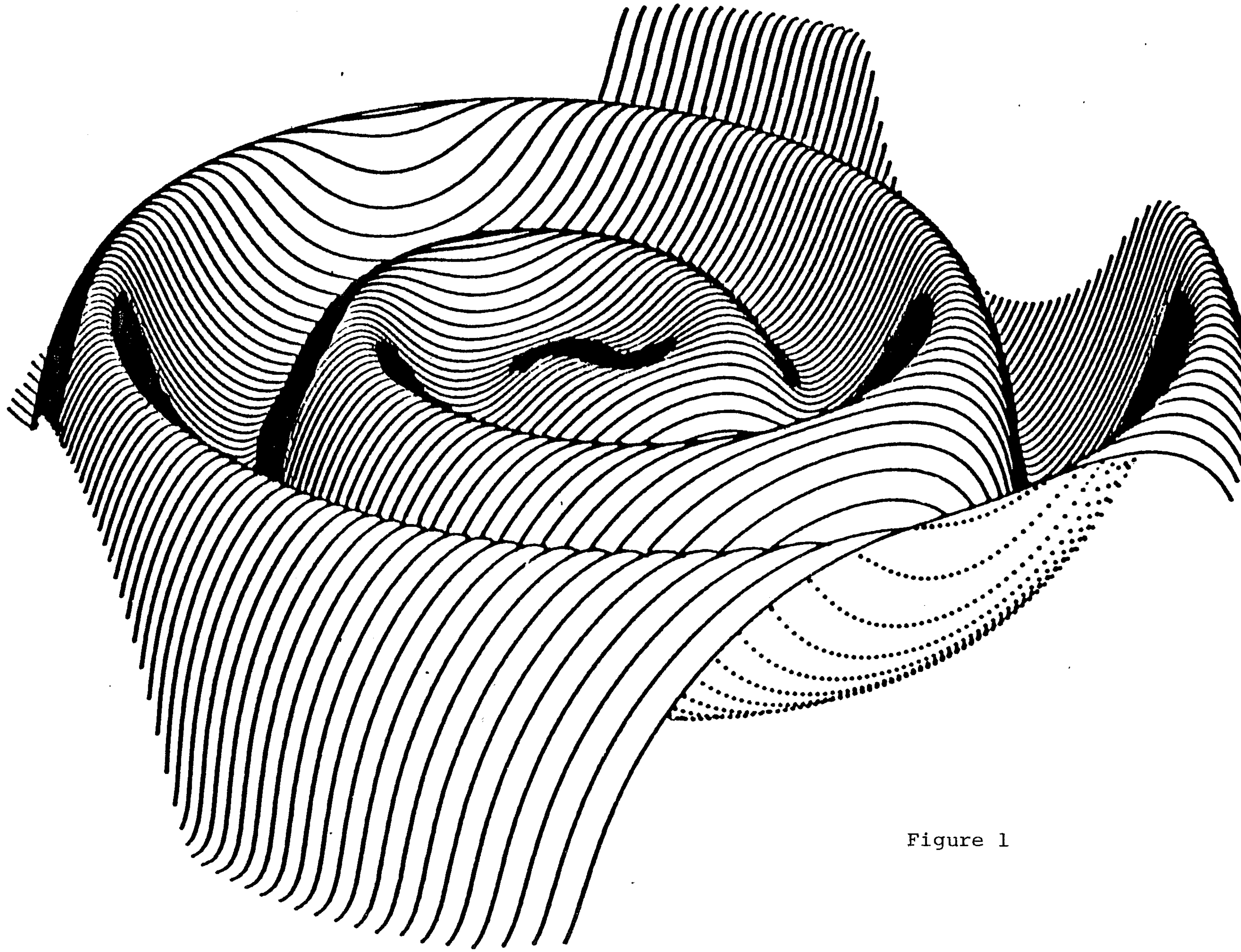


Figure 1

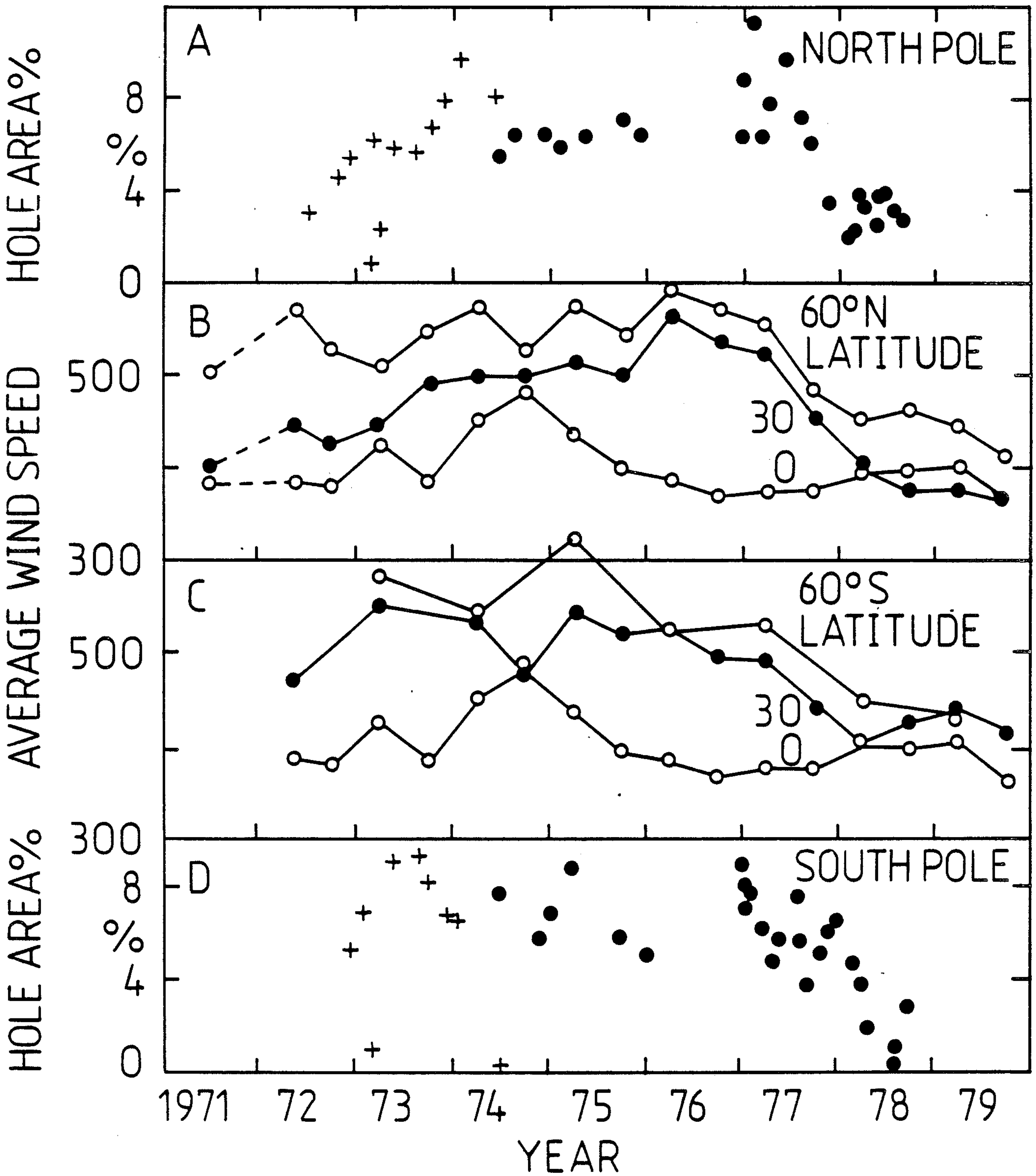

A
NORTH POLE
HOLE AREA%
%
8
4
0
B
60°N LATITUDE
30
0
500
300
AVERAGE WIND SPEED
C
60°S LATITUDE
30
0
500
300
D
SOUTH POLE
HOLE AREA%
%
8
4
0
1971
72
73
74
75
76
77
78
79
YEAR

Figure 2

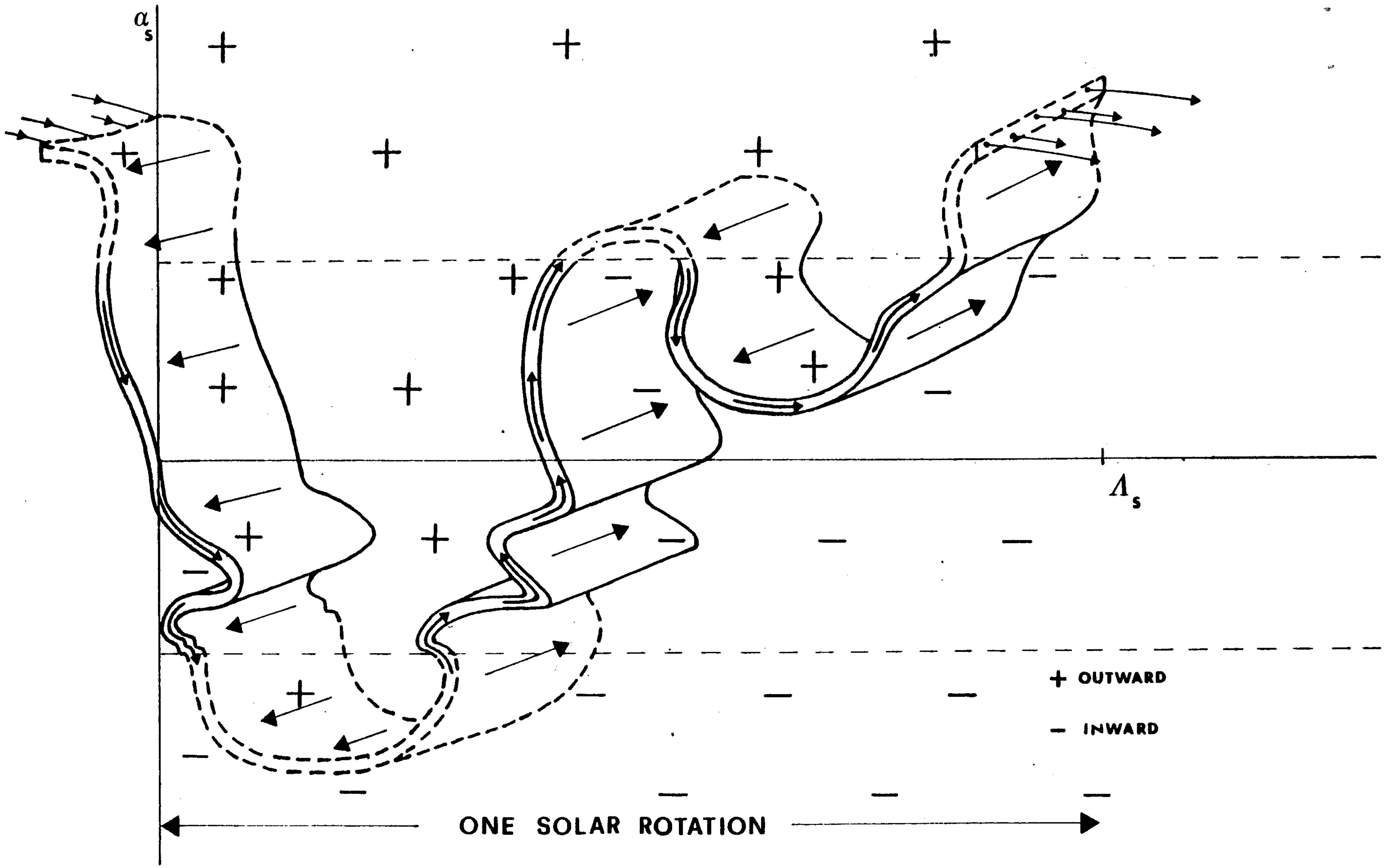


Figure 3

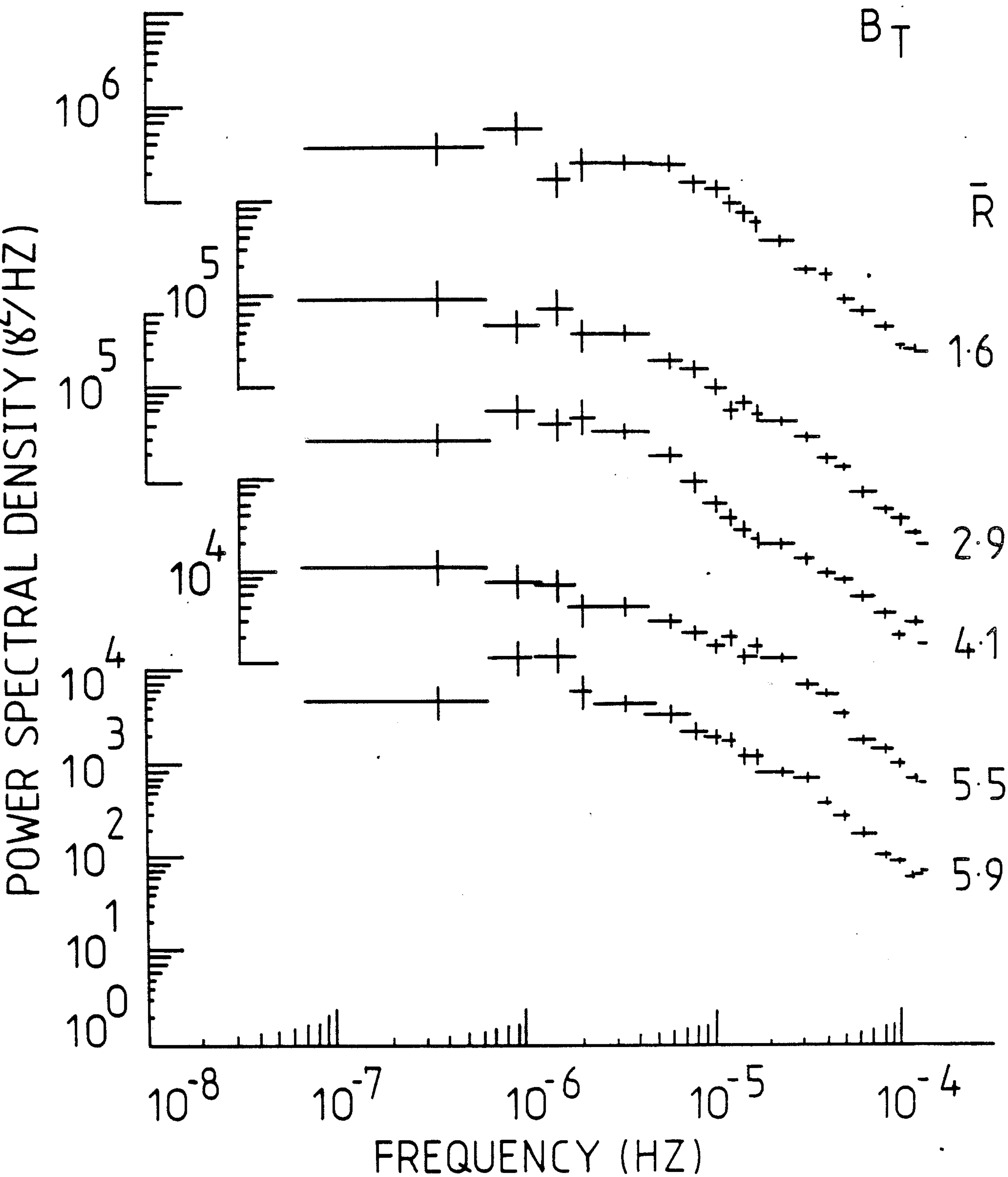

B T
R̄
1·6
2·9
4·1
5·5
5·9
POWER SPECTRAL DENSITY (γ²/HZ)
FREQUENCY (HZ)
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{6}
10^{5}
10^{5}
10^{4}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}

Figure 4

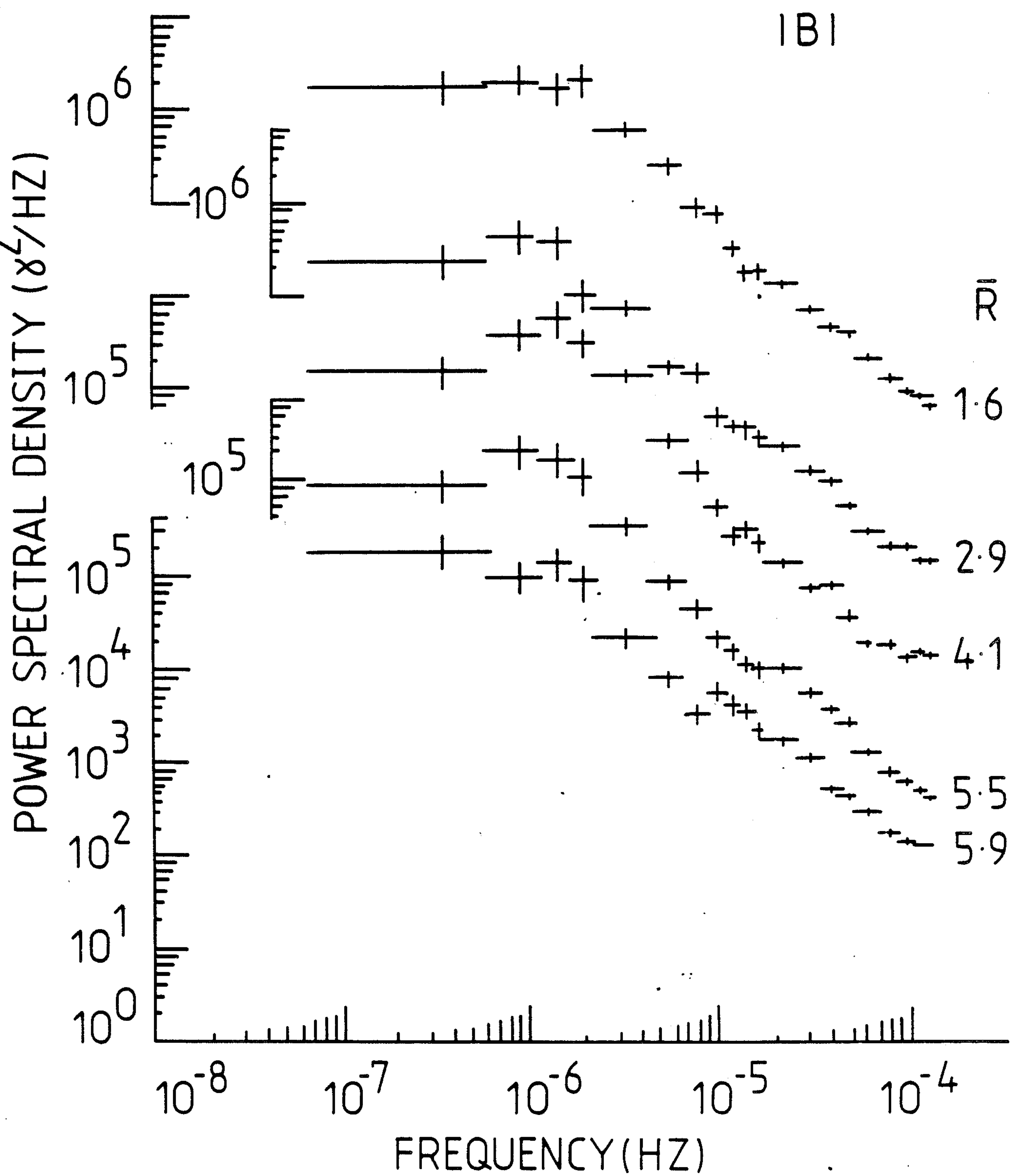

IBI
POWER SPECTRAL DENSITY (γ^2/HZ)
FREQUENCY (HZ)
$\bar{R}$
1·6
2·9
4·1
5·5
5·9
10^6
10^5
10^4
10^3
10^2
10^1
10^0
10^{-8}
10^{-7}
10^{-6}
10^{-5}
10^{-4}

Figure 5

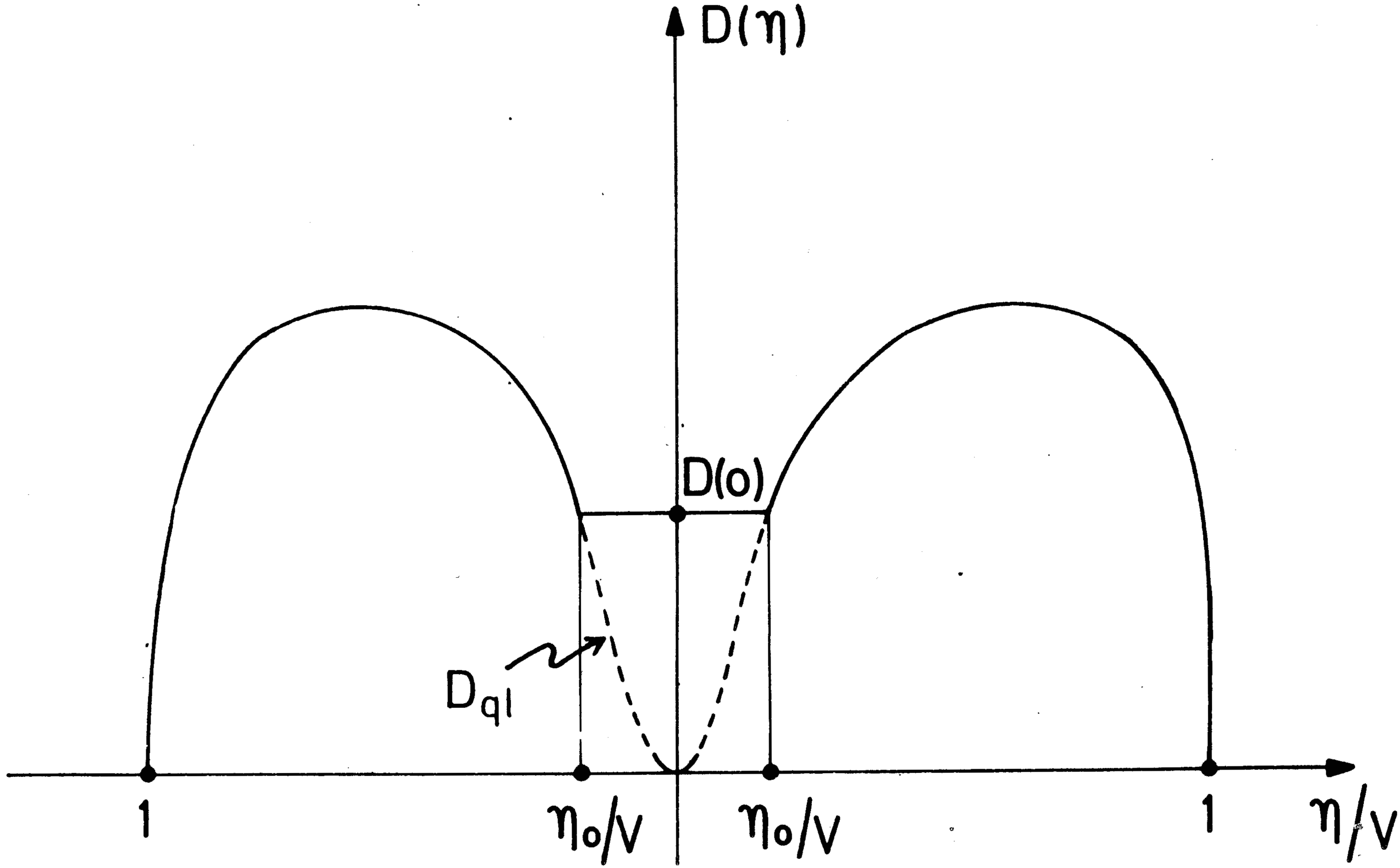


Figure 6

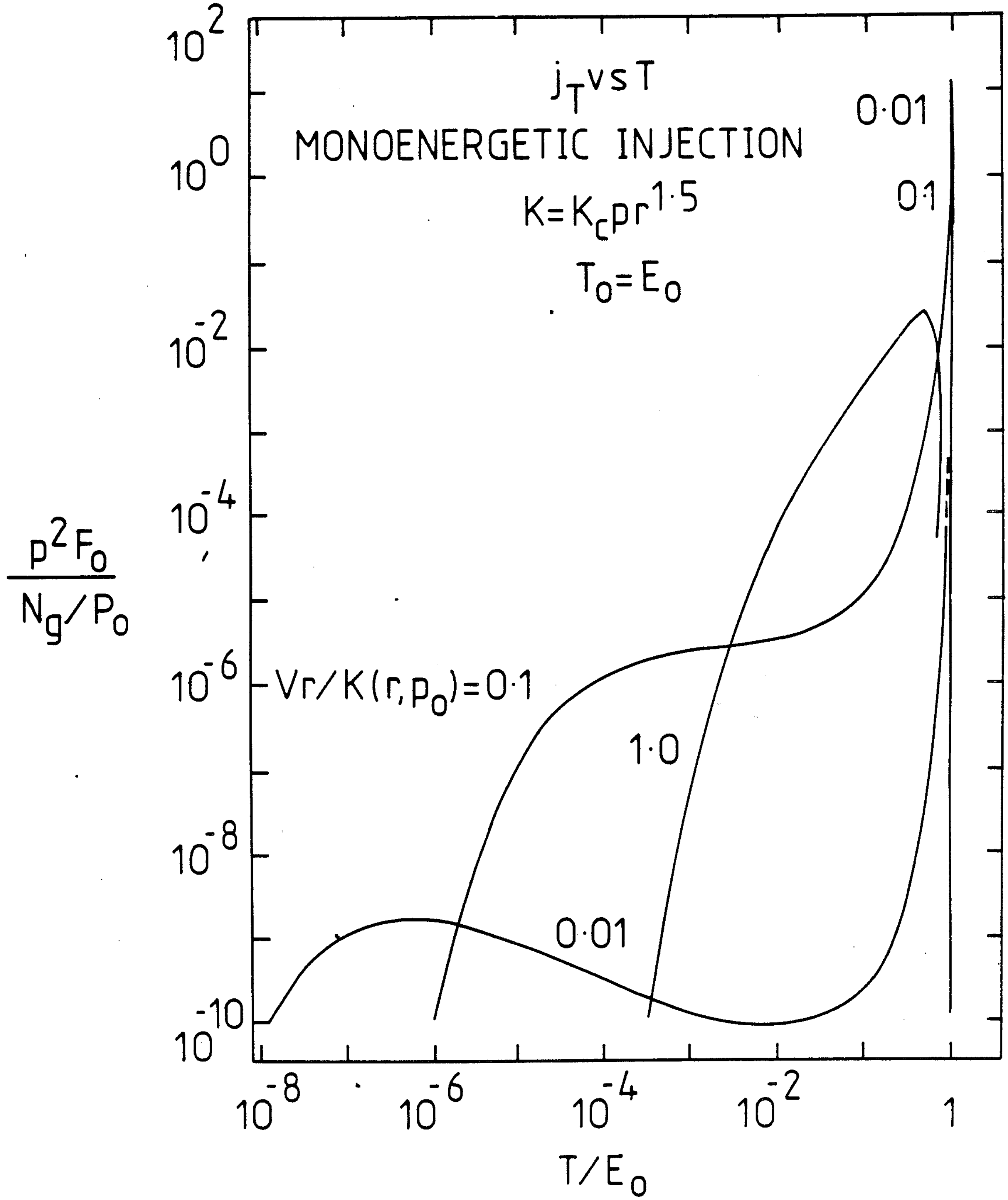

j_T vs T
MONOENERGETIC INJECTION
$K=K_c p r^{1.5}$
$T_0=E_0$
$Vr/K(r,p_0)=0.1$
1·0
0·01
0·01
0·1
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
$\frac{p^2 F_0}{N_g/P_0}$
10^{-8}
10^{-6}
10^{-4}
10^{-2}
1
T/E_0

Figure 7

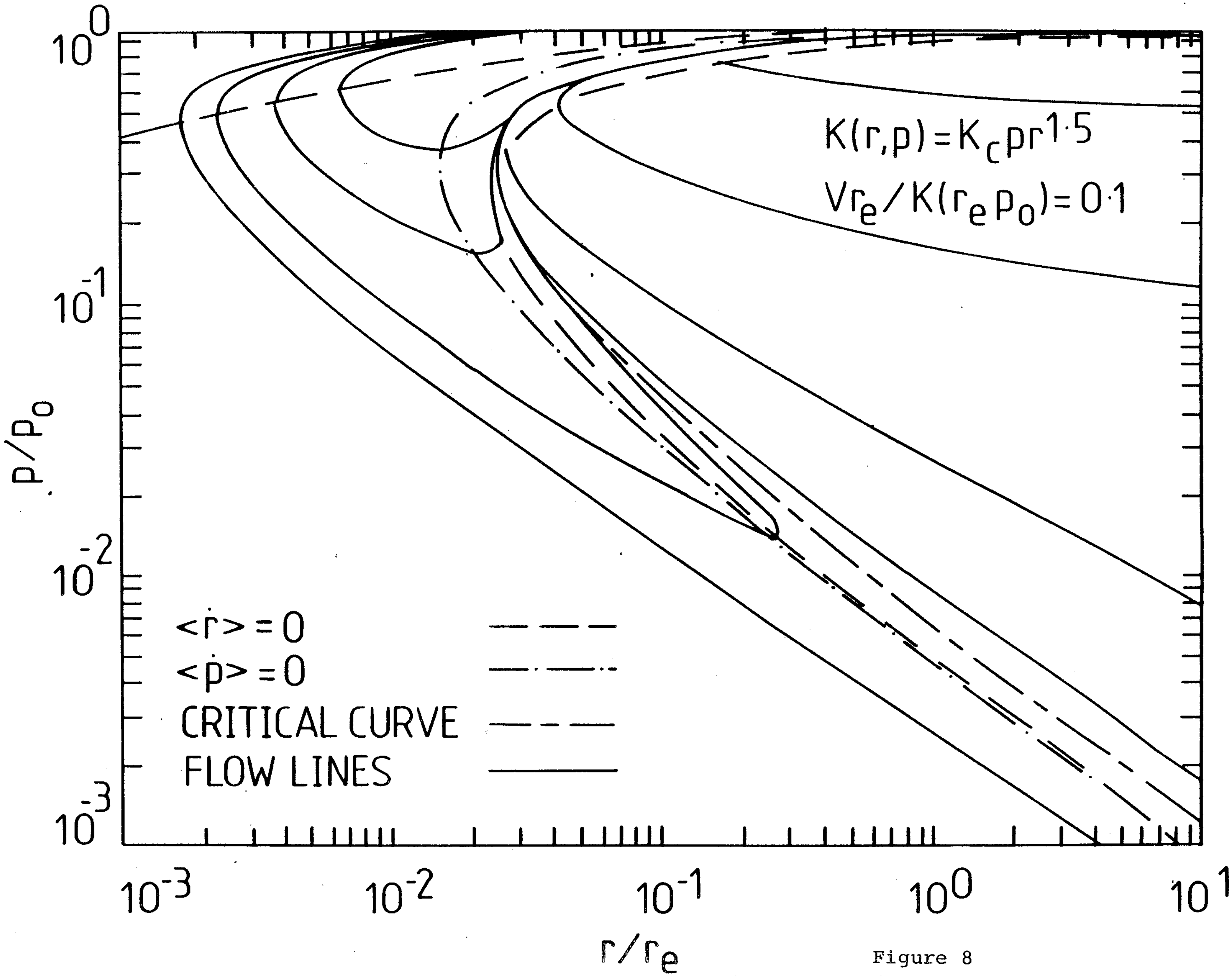

$K(r,p) = K_c p r^{1.5}$
$V r_e / K(r_e p_o) = 0.1$
$<\dot{r}> = 0$
$<\dot{p}> = 0$
CRITICAL CURVE
FLOW LINES
p/p_o
r/r_e
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1

Figure 8

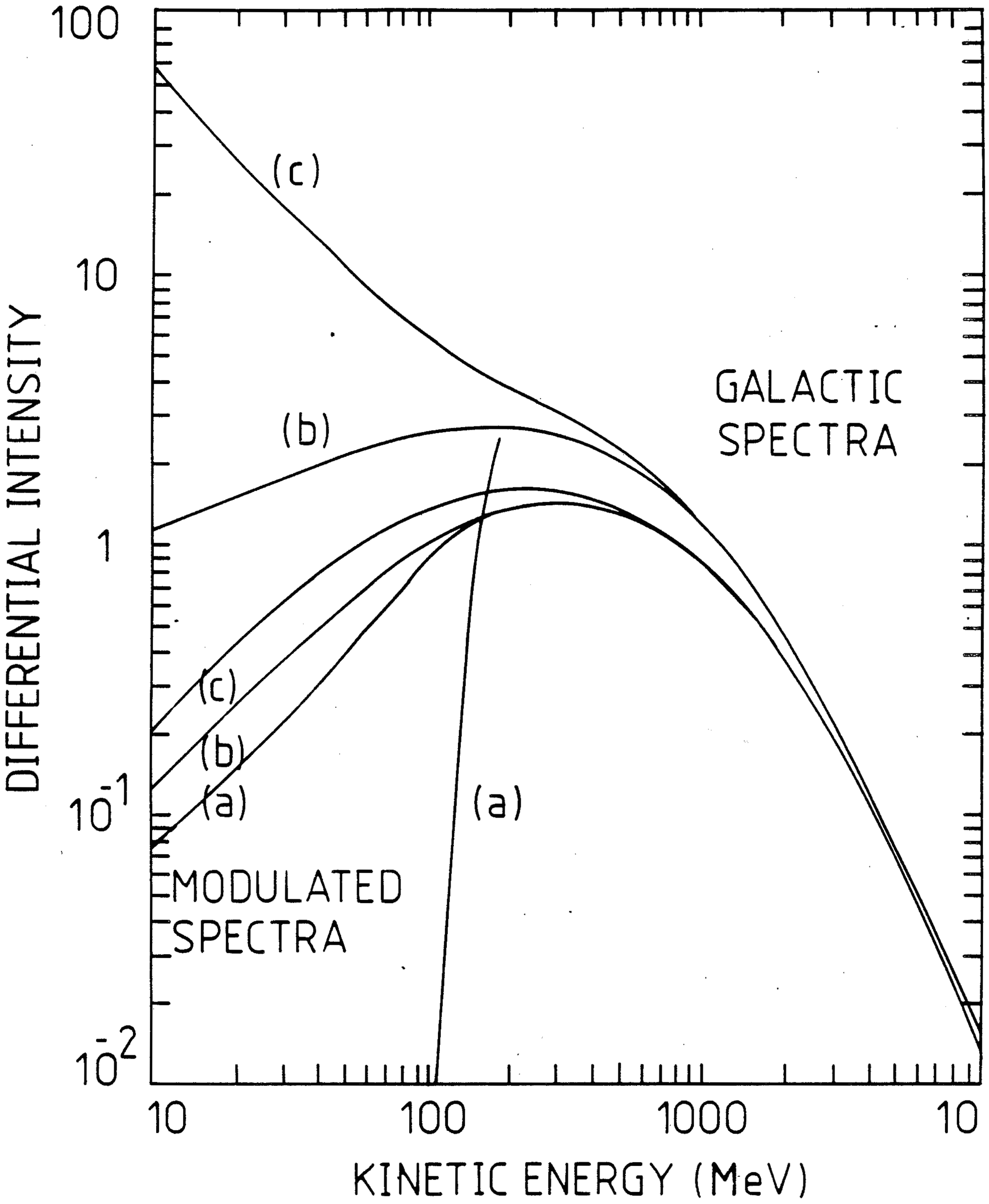
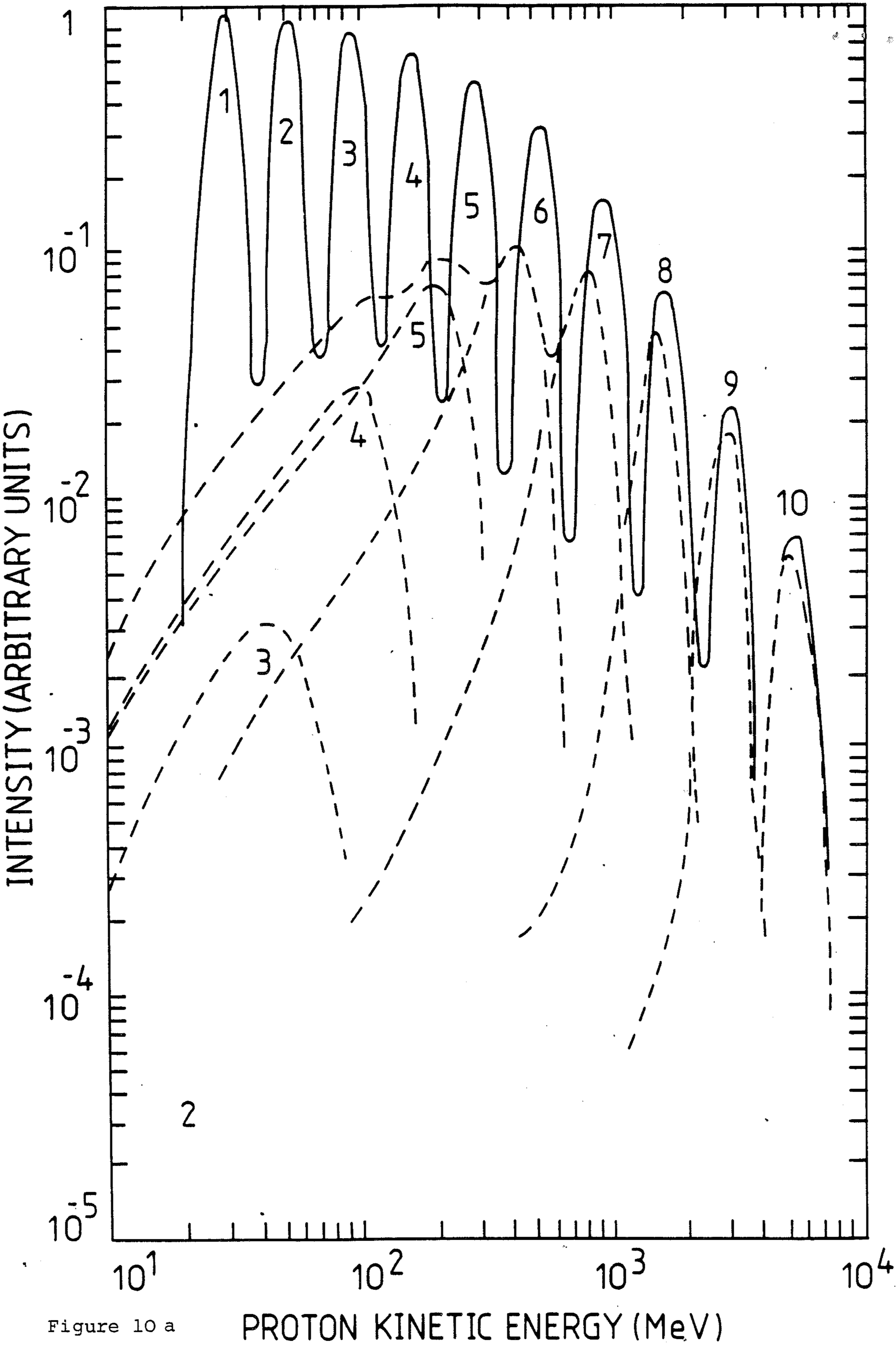
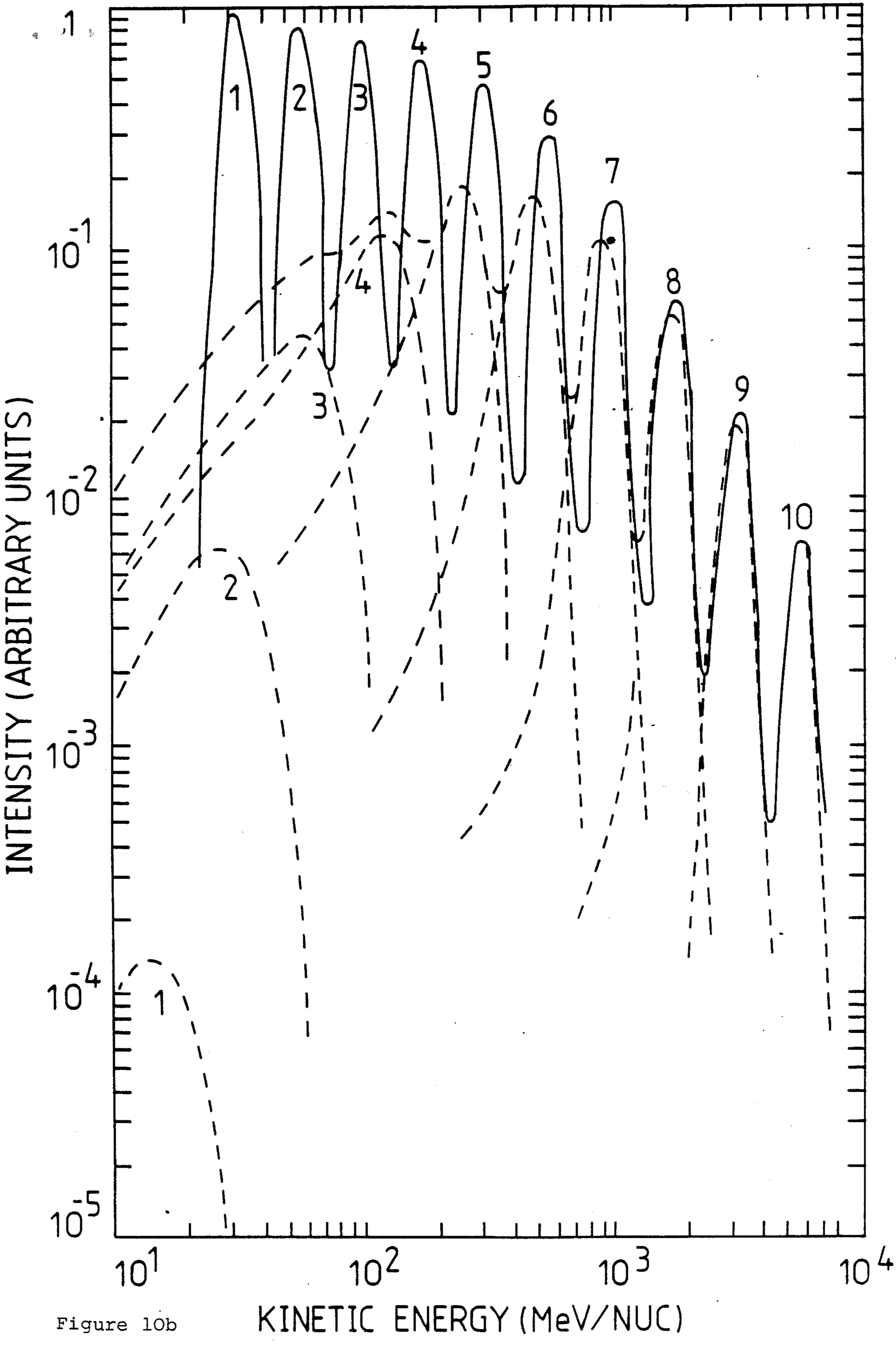

100
10
1
10^{-1}
10^{-2}
DIFFERENTIAL INTENSITY
(c)
(b)
GALACTIC
SPECTRA
(c)
(b)
(a)
(a)
MODULATED
SPECTRA
10
100
1000
10
KINETIC ENERGY (MeV)

Figure 9



1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵
INTENSITY (ARBITRARY UNITS)
10¹
10²
10³
10⁴
PROTON KINETIC ENERGY (MeV)
1
2
3
4
5
6
7
8
9
10
5
4
3
2

Figure 10 a



INTENSITY (ARBITRARY UNITS)
KINETIC ENERGY (MeV/NUC)
.1
10-1
10-2
10-3
10-4
10-5
10 1
10 2
10 3
10 4
1
2
3
4
5
6
7
8
9
10

Figure 10b

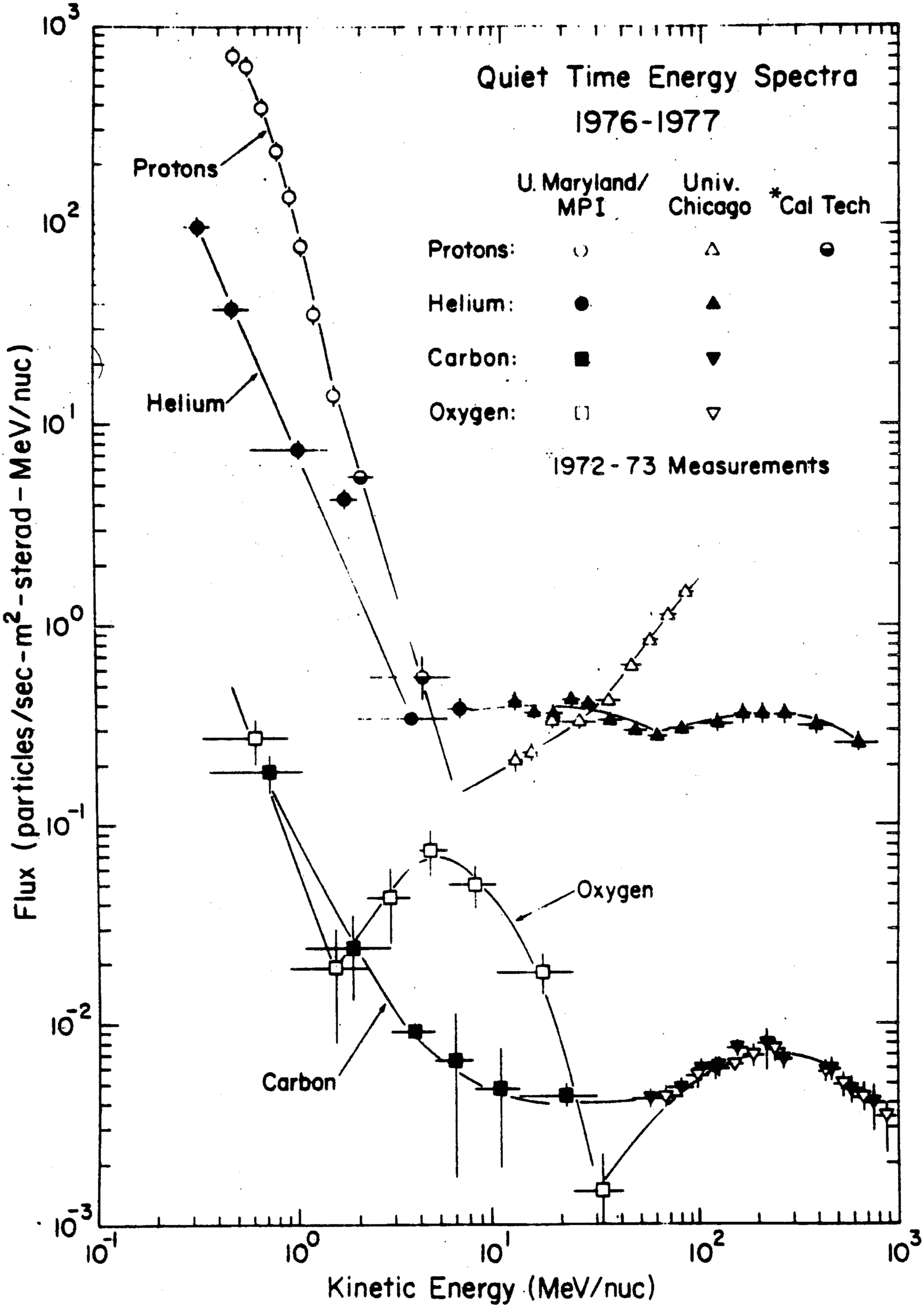

Quiet Time Energy Spectra
1976-1977
U. Maryland/ MPI
Univ. Chicago
*Cal Tech
Protons:
Helium:
Carbon:
Oxygen:
1972-73 Measurements
Protons
Helium
Oxygen
Carbon
Flux (particles/sec-m²-sterad-MeV/nuc)
Kinetic Energy (MeV/nuc)

Figure 11

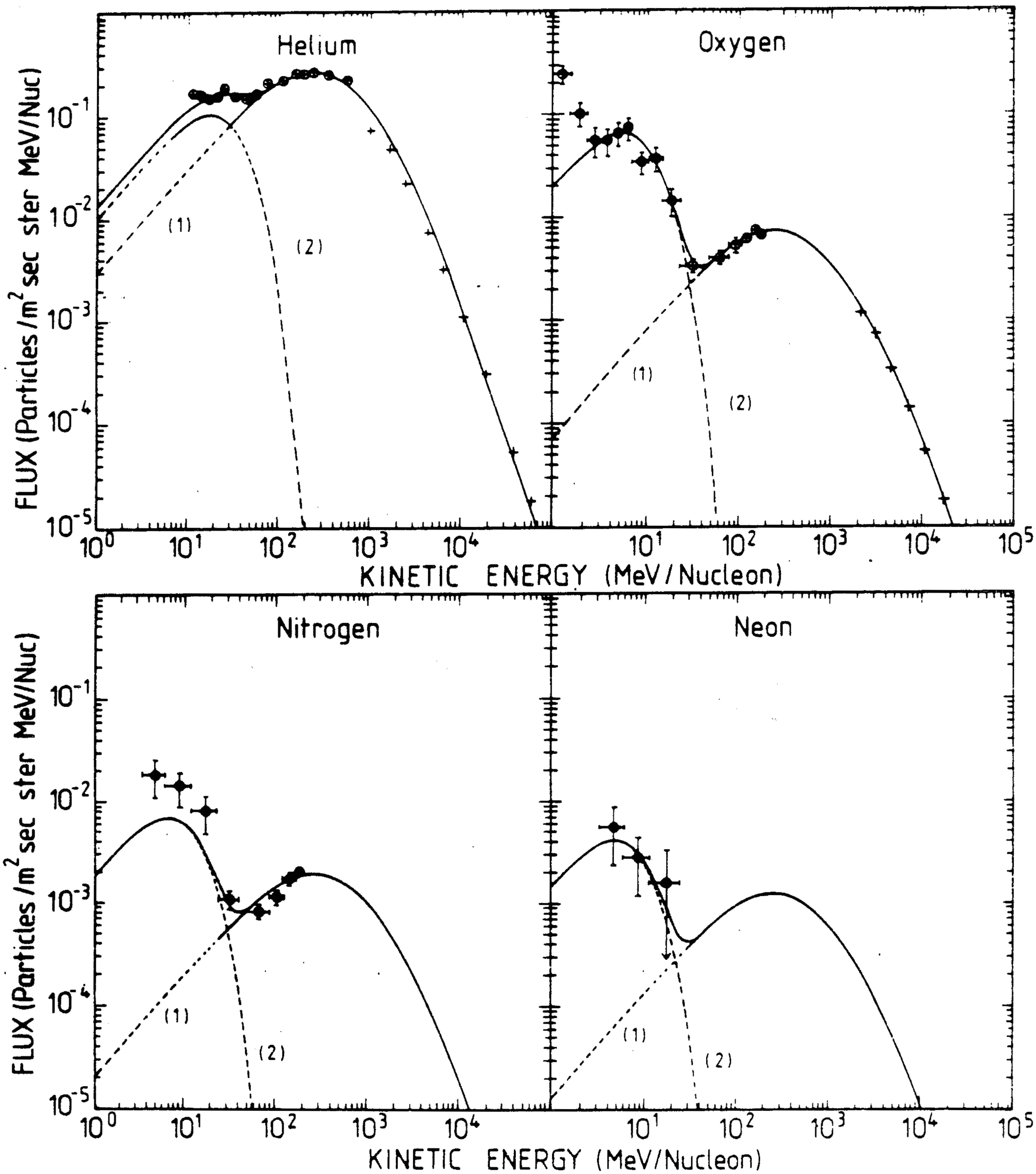

Helium
Oxygen
Nitrogen
Neon
FLUX (Particles/m^2 sec ster MeV/Nuc)
KINETIC ENERGY (MeV/Nucleon)
(1)
(2)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^0
10^1
10^2
10^3
10^4
10^5

Figure 12

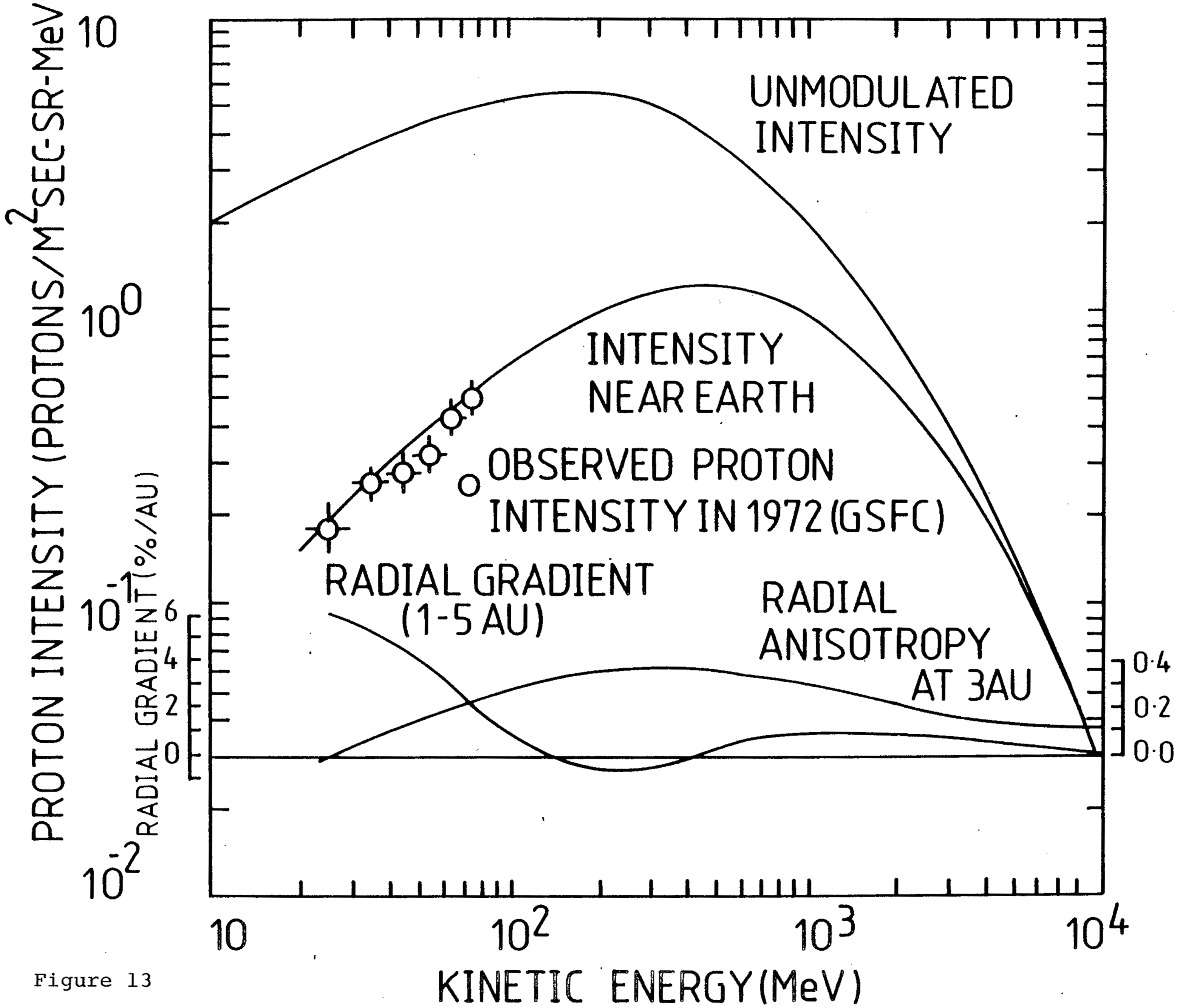

PROTON INTENSITY (PROTONS/M2SEC-SR-MeV
10
10^0
10^-1
10^-2
RADIAL GRADIENT (%/AU)
6
4
2
0
UNMODULATED
INTENSITY
INTENSITY
NEAR EARTH
OBSERVED PROTON
INTENSITY IN 1972 (GSFC)
RADIAL GRADIENT
(1-5 AU)
RADIAL
ANISOTROPY
AT 3AU
0·4
0·2
0·0
10
10^2
10^3
10^4
KINETIC ENERGY (MeV)

Figure 13

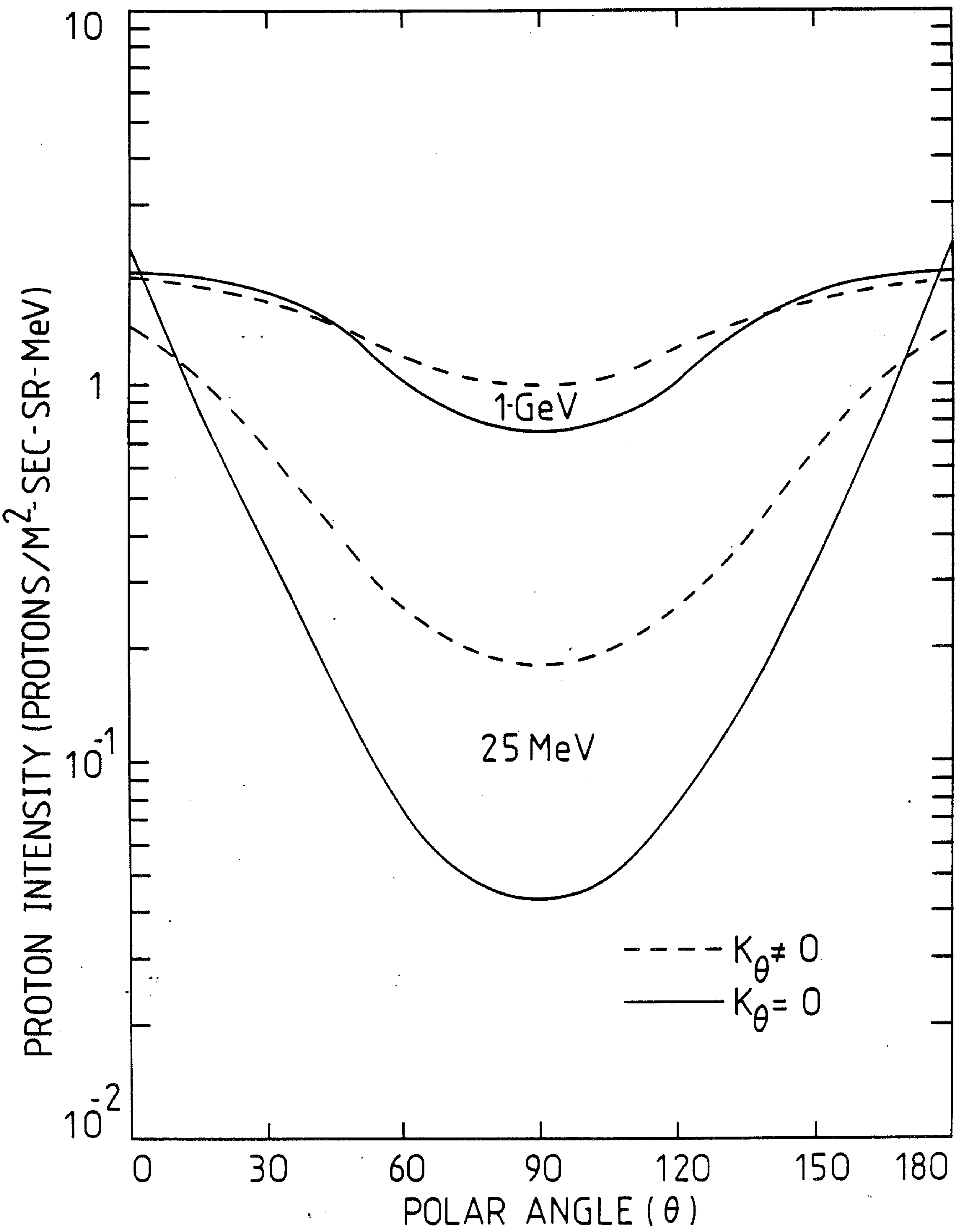

10
1
10^{-1}
10^{-2}
PROTON INTENSITY (PROTONS/M^2-SEC-SR-MeV)
1 GeV
25 MeV
$K_\theta \neq 0$
$K_\theta = 0$
0
30
60
90
120
150
180
POLAR ANGLE (θ)

Figure 14

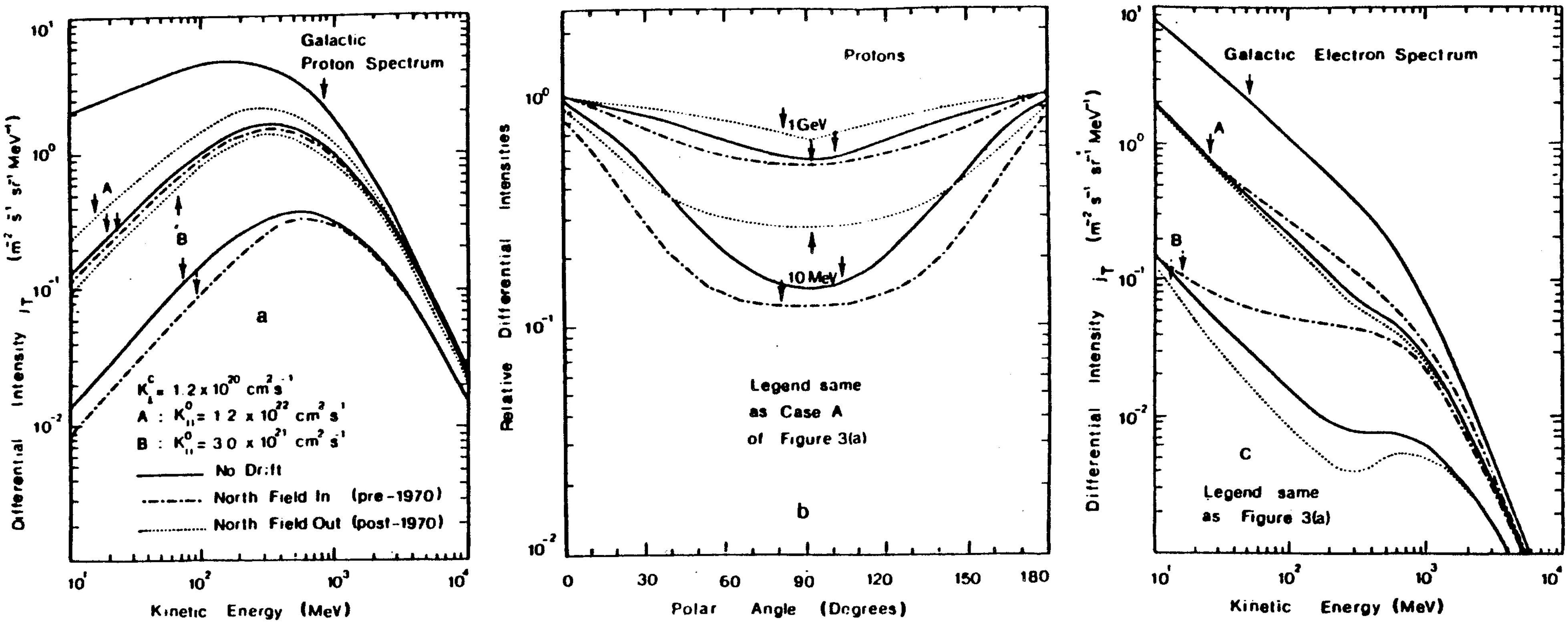
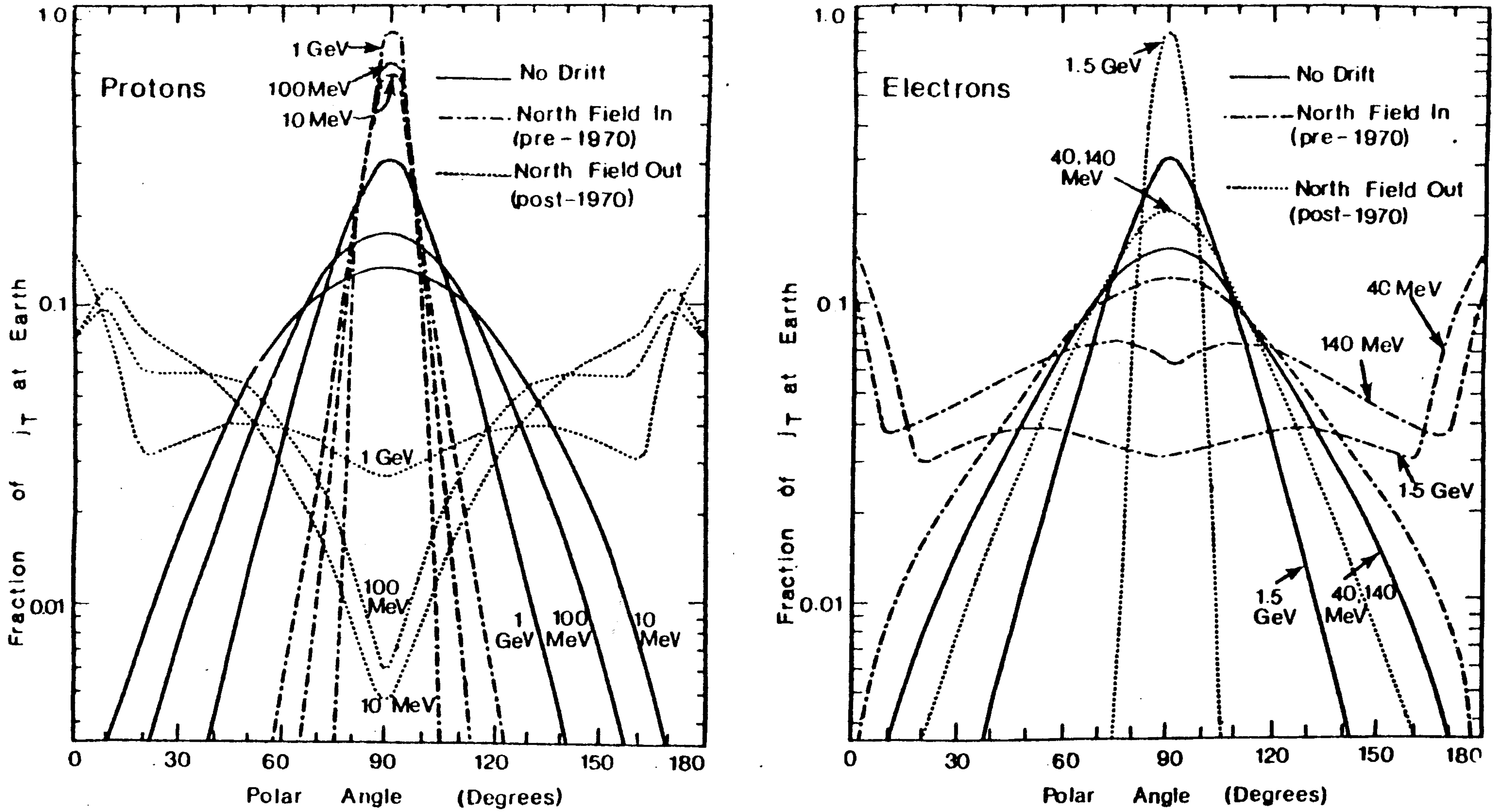

Galactic
Proton Spectrum
Differential Intensity j_T (m^{-2} s^{-1} sr^{-1} MeV^{-1})
A
B
a
$K_\perp^c$ = 1.2 x 10^{20} cm^2s^{-1}
A : $K_{\parallel}^0$ = 1.2 x 10^{22} cm^2 s^{-1}
B : $K_{\parallel}^0$ = 3.0 x 10^{21} cm^2 s^{-1}
No Drift
North Field In (pre-1970)
North Field Out (post-1970)
Kinetic Energy (MeV)
Protons
1GeV
10MeV
Relative Differential Intensities
Legend same
as Case A
of Figure 3(a)
b
Polar Angle (Degrees)
Galactic Electron Spectrum
C
Legend same
as Figure 3(a)
Kinetic Energy (MeV)

Figure 15



Protons
1 GeV
100MeV
10 MeV
No Drift
North Field In (pre-1970)
North Field Out (post-1970)
1 GeV
100 MeV
10 MeV
1 GeV
100 MeV
10 MeV
1.0
0.1
0.01
Fraction of j_T at Earth
0
30
60
90
120
150
180
Polar Angle (Degrees)
Electrons
1.5 GeV
40,140 MeV
No Drift
North Field In (pre-1970)
North Field Out (post-1970)
40 MeV
140 MeV
1.5 GeV
1.5 GeV
40,140 MeV

Figure 16

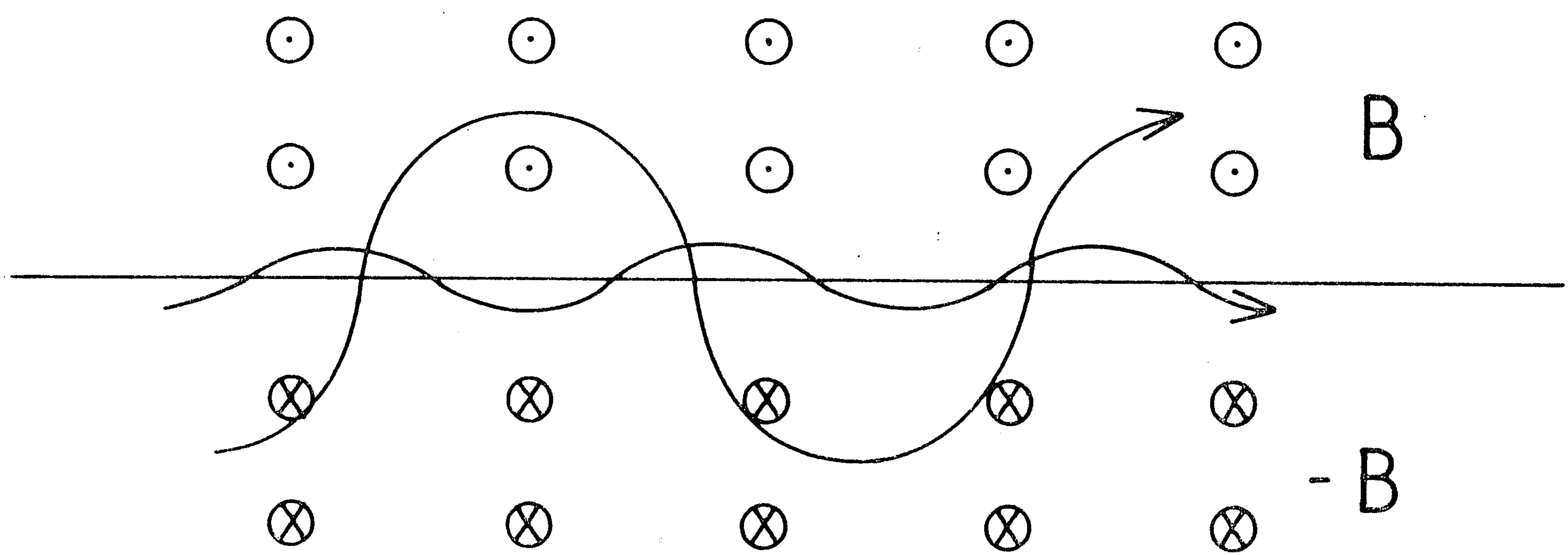


Figure 17

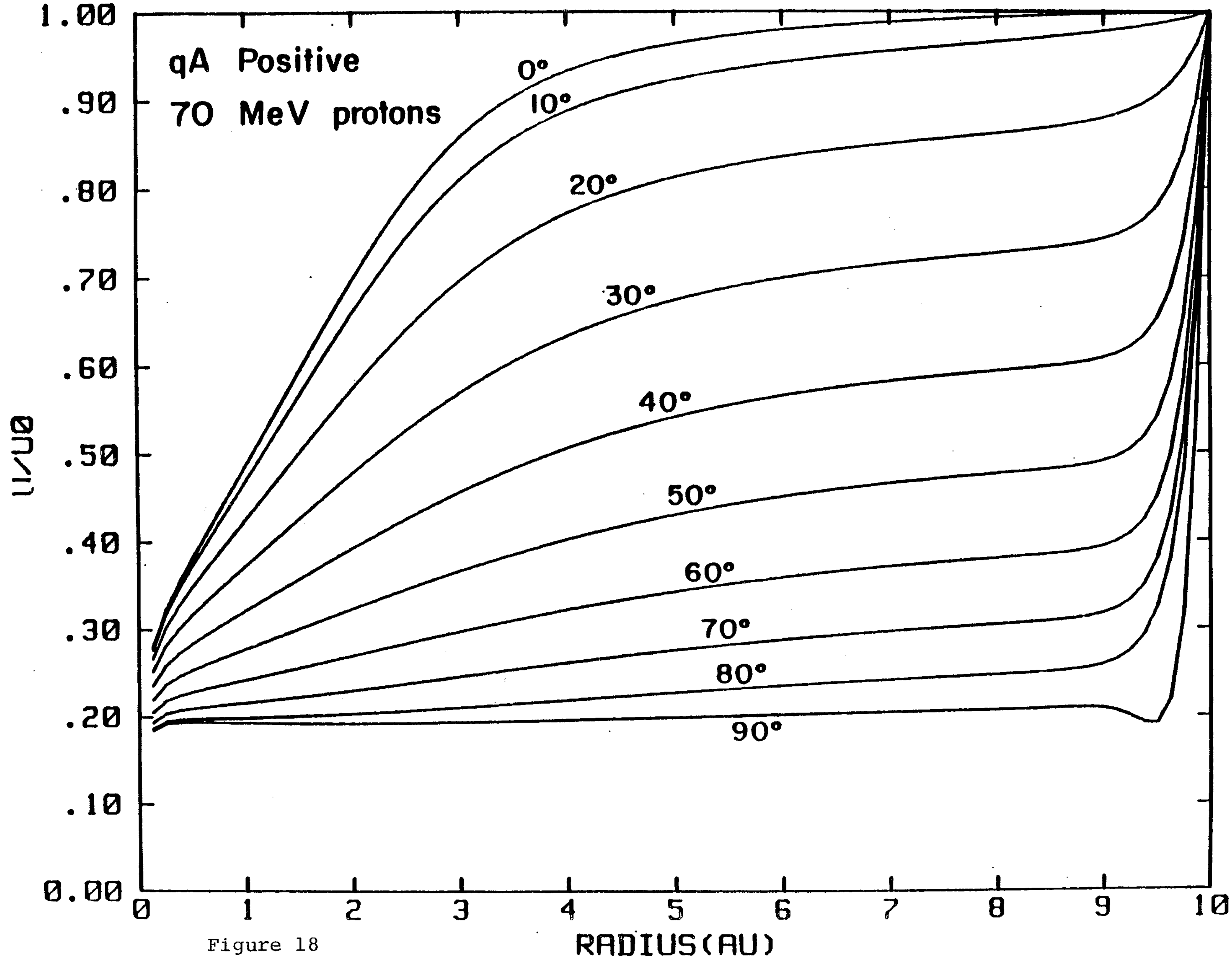
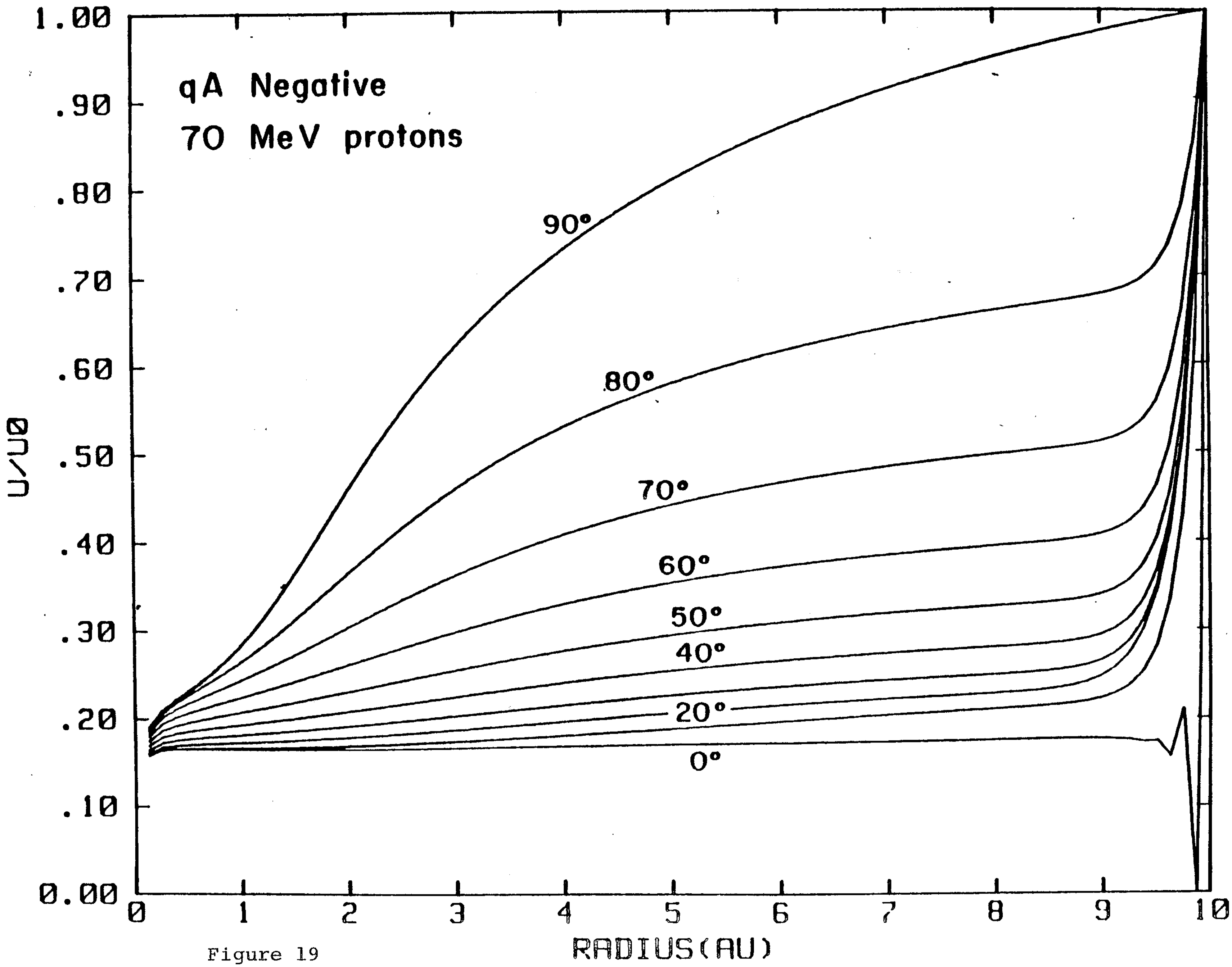

qA Positive
70 MeV protons
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0.00
U/U0
0
1
2
3
4
5
6
7
8
9
10
RADIUS(AU)

Figure 18


qA Negative
70 MeV protons
1.00
.90
.80
.70
.60
.50
.40
.30
.20
.10
0.00
U/U0
90°
80°
70°
60°
50°
40°
20°
0°
0
1
2
3
4
5
6
7
8
9
10
RADIUS(AU)
Figure 19